

الف) $\sqrt{\frac{x-1}{x}} - \frac{x}{x-1}$

$\frac{1-x^2}{x(x-1)} \geq 0$

$x_1 = 0, x_2 = \frac{1}{x}$
 $x_1 = 1$

$(1-\frac{1}{x})$

$D_f = (-\infty, 0) \cup [\frac{1}{x}, 1)$

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ب) $x \neq -2, -1, 0, 1$
 $\frac{x}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow \frac{x(x+2)}{(x-1)(x+2)} \neq 0 \rightarrow x \neq -\frac{2}{x}$
 $D_f = \mathbb{R} - \{-2, -\frac{2}{x}, -1, 0, 1\}$

الف) $\sqrt{((\frac{1}{x})^2 - 9)(x^2 - 2x)}$

$(\frac{1}{x^2} - 9)(x^2 - 2x) \geq 0$

$(\frac{1}{x^2} - 9)(x^2 - 2x) \geq 0$

q, x^2

$x^2, x^2 \rightarrow (x^2)^x$

$x^2 - x^2, 0 \rightarrow -x, x^2$

$x_1 = -2$

$x_2 = 2$

$x_3 = 0$

$x_4 = 1$

$x_5 = \infty$

$x_6 = -\infty$

$x_7 = 1$

$x_8 = 2$

$x_9 = 0$

$x_{10} = -2$

$x_{11} = 2$

$x_{12} = 0$

$x_{13} = 1$

$x_{14} = \infty$

$x_{15} = -\infty$

$x_{16} = 1$

$x_{17} = 2$

$x_{18} = 0$

$x_{19} = -2$

ب) $x-1 \geq 0 \rightarrow x \geq 1$

$\sqrt{y+1} = \sqrt{x-1}$
 $\sqrt{x-1} \leq 2 \rightarrow x-1 \leq 4 \rightarrow x \leq 5$

$D_f = [1, 5]$

$x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow x_1 = -1, x_2 = 2 \rightarrow x \in (-\infty, -1) \cup (2, \infty)$

$x^2 - 1 \geq 0 \rightarrow x \in (-\infty, -1] \cup [1, \infty)$

$\sqrt{x^2 - 1} \geq 1 \rightarrow \sqrt{x^2 - 1} \geq 1 \rightarrow x^2 - 1 \geq 1 \rightarrow x^2 \geq 2 \rightarrow x \geq \sqrt{2} \text{ or } x \leq -\sqrt{2}$

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$D_f = (-\infty, -1) \cup (2, \infty)$

$3 \leq ax - x^2 \leq 0 \rightarrow x^2 - ax + 3 \leq 0 \rightarrow x_1 = -2, x_2 = b$
 $(-2)^2 - a(-2) + 3 = 0 \rightarrow 4 + 2a + 3 = 0 \rightarrow 2a = -7 \rightarrow a = -\frac{7}{2}$
 $a = -\frac{7}{2}$

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$3, x_1, x_2 \rightarrow -\frac{b}{a} \rightarrow a, 1$
 $b, -a \rightarrow 2 - (-\frac{1}{2}) = \frac{1}{2}$
 $-2 \leq b \rightarrow 2 - \frac{1}{2} \leq b \rightarrow b \geq \frac{5}{2}$
 $b \geq a, -\frac{1}{2} \leq \frac{1}{2} \leq \frac{1}{2} \text{ (1)}$

$f(x) - x \geq 0 \rightarrow f(x) \geq x$

$x \geq 1$

$x^2 - 2 \geq x \rightarrow x^2 \geq x + 2 \rightarrow x \geq 1$

$x < 1$

$x^2 + 3 \geq x \rightarrow x^2 \geq x - 3 \rightarrow x \in [-3, 1)$

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$D_f = [-3, \infty)$

$\sin \frac{\pi}{r}, \sqrt{\frac{r}{r}}$ $\cos \frac{\pi}{r} = 1$ $\sqrt{\frac{r}{r}} \leq 1 \Rightarrow \frac{\sqrt{r-r}}{r}$ $r(\frac{\sqrt{r-r}}{r}) = \sqrt{r-r}$ $(x-r) \rightarrow \frac{\sqrt{r-r}}{r} - r, \frac{\sqrt{r-r}}{r}$ $(\sqrt{r-r}) \times (\frac{\sqrt{r-r}}{r})$ $\frac{(\sqrt{r-r})(\sqrt{r-r})}{r} \rightarrow$	$f(x) = rx^2 - \varepsilon x$ $rx(x-r)$ $r - \varepsilon = -1$ $\frac{-1}{r}$	6
$f(a) = \frac{a-1}{a-1}$ $f(f(a)) \rightarrow \frac{f(a)-1}{f(a)-1} = \frac{\frac{a-1}{a-1}-1}{\frac{a-1}{a-1}-1} \rightarrow \frac{\frac{a-1}{a-1}-1}{\frac{a-1}{a-1}-1} \rightarrow \frac{r}{a-1}$ $\frac{ra}{a-1}, \frac{ra}{r} = a$ $a-1 \neq 0$ $a \neq 1$ $f(a) \neq 1 \Rightarrow \frac{a-1}{a-1} \neq 1 \rightarrow a-1 \neq a-1 \rightarrow 1 \neq -1 \checkmark$	$\frac{ra}{a-1}, \frac{ra}{r} = a$ $\frac{ra}{a-1}, \frac{ra}{r} = a$ $a \in \mathbb{R} \setminus \{1\}$	7
$f(g(a)) \rightarrow (g(a))^r - \varepsilon g(a) = -\varepsilon$ $(g(a))^r - \varepsilon g(a) + \varepsilon = 0$ $(g(a)-r)^r = 0$ $g(a)-r = 0 \rightarrow g(a) = r$ $\sqrt{a^2-1} = r \Rightarrow a^2 = 1 + \varepsilon$ $a^2, \delta \rightarrow a, \pm \sqrt{\varepsilon}$ $a = \sqrt{\varepsilon}$ $a = -\sqrt{\varepsilon}$		8
$f(-r)$ $x \in \mathbb{R}, -r \rightarrow x = -r$ $f(x)$ $x \in \mathbb{R}, \varepsilon$ $x = 1$ $f(x) = 1 - \delta(1) + \gamma, \gamma$ $\forall x \in \mathbb{R} \rightarrow 1 \pm \varepsilon$	$f(-r) = (-r)^r - \delta(-r) + \gamma = \gamma$ $\forall x \in \mathbb{R}, -r \rightarrow x = -r$	9
$(\varepsilon \cos^2 x + \delta \cos x, r, \delta, \frac{\gamma}{r}) \leq r$ $\wedge \cos^2 x \leq 1 \wedge \cos x = -1 \Rightarrow \Delta = \sqrt{r^2 \varepsilon} = 1 \wedge$ $(\cos^2 x + \delta \cos x) \leq r$ $\tan^2 x = \frac{1}{\cos^2 x} - 1 \rightarrow \cos x = \frac{1}{r}$	$\frac{-1 \pm 1 \wedge}{r(1)} = \frac{-1 \pm 1 \wedge}{1 \gamma} =$ $\textcircled{1} = \frac{1}{r} \checkmark$ $\textcircled{2} = -1, \sqrt{\delta} = \frac{\gamma}{1 \gamma}$ $\leftarrow x$ $-1 \leq \cos x \leq 1$	$\frac{1}{\varepsilon}$ $\frac{1}{\varepsilon} = 1, \varepsilon = 1, \textcircled{1}$ $f(r, \delta) \textcircled{1}$