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$$f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} = \sqrt{\frac{(x-1)^2 - x^2}{x(x-1)}} = \sqrt{\frac{1-2x}{x(x-1)}} \Rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, \infty)$$

$$f(x) = \frac{1}{x+1} - \frac{x}{x-1} \Rightarrow \left. \begin{array}{l} ① x \neq -1 \text{ و } 1 \\ ② \frac{x-2x-2}{(x+1)x} = \frac{(x+2)(x-2)}{x(x+1)(x+5)} \end{array} \right\} \Rightarrow D_f = \mathbb{R} - \{-1, -\frac{5}{2}, 1\}$$

$$f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^x - 9\right) \left(3x - \frac{1}{x}\right)} \Rightarrow D_f = \mathbb{R} - (-2, 2, 5)$$

$$\sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \left. \begin{array}{l} ① x \geq 1 \\ ② y \geq -1 \end{array} \right\} \Rightarrow (y+1)^2 = 11 - x + 1 \Rightarrow 12 - x \geq 0 \Rightarrow x \leq 12 \Rightarrow D_f = [1, 12]$$

$$f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \Rightarrow \left. \begin{array}{l} ① x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0 \Rightarrow x < -1 \text{ or } x > 2 \\ ② x^2 - 1 > 0 \Rightarrow x < -1 \text{ or } x > 1 \\ ③ \sqrt{x^2 - 1} \neq -1 \end{array} \right\} \Rightarrow D = (-\infty, -1) \cup (2, \infty)$$

$$f(x) = \sqrt{3 + \alpha x - x^2} \Rightarrow D_f = [-2, b] \Rightarrow \left. \begin{array}{l} \text{بافتتاحیه مفتی بودن} \\ \text{یعنی عبارت دارای دو ریشه ۲ و ۳ بوده است} \end{array} \right\} \Rightarrow \alpha + b = \frac{-1}{2} + \frac{3}{2} = 1$$

$$f(x) = \begin{cases} 3x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases} \Rightarrow \left. \begin{array}{l} ① x \geq 1 \Rightarrow \sqrt{3x-2-x} = \sqrt{2x-2} \Rightarrow 2x-2 \geq 0 \Rightarrow x \geq 1 \\ ② x < 1 \Rightarrow \sqrt{2x+3-x} = \sqrt{x+3} \Rightarrow x+3 \geq 0 \Rightarrow x \geq -3 \end{array} \right\} \Rightarrow D_f = [-3, \infty)$$

$$f(x) = 2x^2 - 4x \xrightarrow{x = \frac{\sqrt{3}+1}{2}} \frac{\sqrt{3}+2}{2} \left(\frac{2\sqrt{3}+4}{2} - 4 \right) = \frac{(\sqrt{3}+2)(\sqrt{3}-2)}{2} = \frac{3-4}{2} = \frac{-1}{2}$$

$$f\left(\sin \frac{\pi}{4} + \cos 2\pi\right) = f\left(\frac{\sqrt{2}}{2} + 1\right) = \frac{2\sqrt{2}+4}{2} \left(\frac{2\sqrt{2}+4}{2} - 4 \right) = \frac{(\sqrt{2}+2)(\sqrt{2}-2)}{2} = \frac{2-4}{2} = \frac{-2}{2} = -1$$

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$$\left. \begin{aligned} f(x) &= \frac{x+1}{x-1} \\ f(f(x)) &= x \end{aligned} \right\} \Rightarrow f(a) = \frac{a+1}{a-1} \Rightarrow f\left(\frac{a+1}{a-1}\right) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{a+1+a-1}{a+1-a+1} = \frac{2a}{2} = a$$

a می تواند هر مقداری باشد به جز $a=1$

-A

$$\left. \begin{aligned} f(x) &= x^2 - \epsilon x \\ g(x) &= \sqrt{x^2 - 1} \end{aligned} \right\} \Rightarrow -\epsilon = x^2 - \epsilon x \Rightarrow x^2 - \epsilon x + \epsilon = 0 \Rightarrow (x-2)^2 = 0 \Rightarrow x=2$$

می توان $\epsilon = \sqrt{a^2 - 1}$ یا $\epsilon = -\sqrt{a^2 - 1}$ $a^2 - 1 = \epsilon = \sqrt{a^2 - 1}$

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$$f(x+y) = x^2 - \omega x + y \Rightarrow \left. \begin{aligned} \textcircled{1} x=1 \Rightarrow f(\epsilon) &= 1 - \omega(1) + \epsilon = 2 \\ \textcircled{2} x=-2 \Rightarrow f(-3) &= 4 - \omega(-2) + \epsilon = 12 \end{aligned} \right\} \Rightarrow f(\epsilon) \times f(-3) = 2 \times 12 = 24$$

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$$f(\epsilon \cos^2 x + \omega \cos x) = \tan^2 x \Rightarrow \epsilon \cos^2 x + \omega \cos x = \frac{1}{\epsilon} \Rightarrow \epsilon \cos^2 x + \omega \cos x - \frac{1}{\epsilon} = 0 \Rightarrow (\epsilon \cos x - 1)(\epsilon \cos x + \frac{\omega}{\epsilon}) = 0$$

در ازای $\frac{1}{\epsilon}$ $\cos x = \frac{1}{\epsilon}$ $\Rightarrow \sin x = \pm \sqrt{1 - \frac{1}{\epsilon^2}}$ و $\cos x = \frac{1}{\epsilon}$

در هر صورت $1 + \tan^2 x = \frac{1}{\cos^2 x} \Rightarrow 1 + \tan^2 x = \epsilon \Rightarrow \tan^2 x = \epsilon - 1 = f(\frac{1}{\epsilon}, \omega)$

چون باید در بازه $[-1, 1]$ باشد $f(\frac{1}{\epsilon}, \omega)$