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$$f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} = \sqrt{\frac{(x-1)^2 - x^2}{x(x-1)}} = \sqrt{\frac{1-2x}{x(x-1)}} \Rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$$

$$f(x) = \frac{1}{x+1} - \frac{x}{x-1} \Rightarrow \left. \begin{array}{l} ① x \neq -1 \text{ و } 1 \\ ② \frac{x-2x-2}{(x+1)x} = \frac{(x+2)(x-2)}{x(x+1)(x+5)} \end{array} \right\} \Rightarrow D_f = \mathbb{R} - \left\{ -1, -\frac{5}{2}, 1 \right\}$$

$$f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^x - 9\right) \left(\frac{1}{x} - 9\right)} \Rightarrow D_f = \mathbb{R} - (-2, 2, \infty)$$

$$\sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \left. \begin{array}{l} ① x \geq 1 \\ ② y \geq -1 \end{array} \right\} \Rightarrow D_f = [1, 12]$$

$$f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \Rightarrow \left. \begin{array}{l} ① x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0 \Rightarrow x < -1 \text{ or } x > 2 \\ ② x^2 - 1 > 0 \Rightarrow x < -1 \text{ or } x > 1 \\ ③ \sqrt{x^2 - 1} \neq -1 \end{array} \right\} \Rightarrow D_f = \mathbb{R} - [-1, 2]$$

$$f(x) = \sqrt{3 + \alpha x - x^2} \Rightarrow D_f = [-2, b] \Rightarrow \alpha + b = \frac{-1}{2} + \frac{3}{2} = 1$$

$$f(x) = \begin{cases} 3x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases} \Rightarrow D_f = [-3, +\infty)$$

$$f(x) = 2x^2 - 4x \Rightarrow f\left(\frac{\sqrt{3}+1}{2}\right) = \frac{3-4}{2} = \frac{-1}{2}$$

-V

$$\left. \begin{aligned} f(x) &= \frac{x+1}{x-1} \\ f(f(x)) &= x \end{aligned} \right\} \Rightarrow f(a) = \frac{a+1}{a-1} \Rightarrow f\left(\frac{a+1}{a-1}\right) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{a+1+a-1}{a+1-a+1} = \frac{2a}{2} = a$$

a می تواند هر مقداری باشد بجز $a=1$

-A

$$\left. \begin{aligned} f(x) &= x^2 - \epsilon x \\ g(x) &= \sqrt{x^2 - 1} \end{aligned} \right\} \Rightarrow -\epsilon = x^2 - \epsilon x \Rightarrow x^2 - \epsilon x + \epsilon = 0 \Rightarrow (x-2)^2 = 0 \Rightarrow x=2$$

می توانیم $\epsilon = \sqrt{a^2 - 1}$ بگذاریم

$$a^2 - 1 = \epsilon \Rightarrow \boxed{a = \pm \sqrt{\epsilon + 1}}$$

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$$f(x+y) = x^2 - \omega x + y \Rightarrow \left. \begin{aligned} ① x=1 \Rightarrow f(\epsilon) &= 1 - \omega(1) + \epsilon = 2 \\ ② x=-2 \Rightarrow f(-3) &= 4 - \omega(-2) + \epsilon = 12 \end{aligned} \right\} \Rightarrow \boxed{f(\epsilon) \times f(-3) = 2 \times 12 = 24}$$

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$$f(\epsilon \cos^2 x + \omega \cos x) = \tan^2 x \Rightarrow \epsilon \cos^2 x + \omega \cos x = \frac{1}{\epsilon} \Rightarrow \epsilon \cos^2 x + \omega \cos x - \frac{1}{\epsilon} = 0 \Rightarrow (\epsilon \cos x - 1)(\epsilon \cos x + \frac{1}{\epsilon}) = 0$$

در ازای $\frac{1}{\epsilon}$ $\cos x = \frac{1}{\epsilon}$ $\Rightarrow \sin x = \pm \sqrt{1 - \frac{1}{\epsilon^2}}$ و $\cos x = \frac{1}{\epsilon}$

در هر صورت $1 + \tan^2 x = \frac{1}{\cos^2 x} \Rightarrow 1 + \tan^2 x = \epsilon \Rightarrow \boxed{\tan^2 x = \epsilon - 1} = f\left(\frac{1}{\epsilon}, \omega\right)$

چون باید در بازه $[-1, 1]$ باشد $f\left(\frac{1}{\epsilon}, \omega\right)$