

۲. افین عالی نورستی

الف) $\frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{-2x+1}{x(x-1)} \geq 0$ $\frac{x}{y} + \frac{0}{\frac{1}{x}} - \frac{1}{\frac{1}{x}} + \frac{1}{\frac{1}{x}} - D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $\frac{x+\Delta}{(x-1)(x-\frac{1}{x})} \neq 0$ $x+\Delta \neq 0$ $x \neq 1$ $x \neq 0$ $x \neq -\frac{\Delta}{x}$ $x \neq 1$ $x \neq -2$ $D_f = \mathbb{R} - \{0, \pm 1, -\frac{\Delta}{x}, -\frac{\Delta}{x}\}$

الف) $(x^2 - x)(x^2 - 2x) \geq 0$ $\frac{x}{y} + \frac{-2}{\frac{1}{x}} - \frac{\Delta}{\frac{1}{x}} + D_f = (-\infty, -2] \cup [\frac{\Delta}{x}, +\infty)$

ب) I) $x-1 \geq 0$ $x \geq 1$ $D_f = [1, +\infty)$ II) $0 \leq \sqrt{x-1} \leq 2 \rightarrow \sqrt{y+1} = 2 - \sqrt{x-1}$ $0 \leq x \leq 17$

I) $x^2 - 1 \geq 0$ $x^2 \geq 1$ $x = (-\infty, -1] \cup [1, +\infty)$ II) $\sqrt{x^2 - 1} \neq -1 \rightarrow \mathbb{R}$ III) $x^2 - x - 2 > 0$ $(x-2)(x+1) > 0$ $x = (-\infty, -1] \cup [2, +\infty)$ $D_f = I \cap II \cap III = (-\infty, -1] \cup [2, +\infty)$

$x^2 - 2a - 1 = 0$ $2a = -1$ $a = -\frac{1}{2}$ $f(x) = \sqrt{-(x^2 + \frac{1}{x} - 2)}$ $\frac{x}{y} - \frac{-2}{\frac{1}{x}} + \frac{\frac{1}{x}}{\frac{1}{x}} - D_f = [-2, \frac{1}{x}] \rightarrow b = \frac{1}{x}$

$a + b = \frac{1}{x} - \frac{1}{2} = 1$

$f(x) - x \geq 0$

I) $x \geq 1 \rightarrow f(x) - x = x^2 - x - x = x^2 - 2x$ $x^2 - 2x \geq 0 \rightarrow x \geq 1$ II) $x < 1 \rightarrow f(x) - x = x^2 + 3 - x = x^2 - x + 3$ $x^2 - x + 3 \geq 0 \rightarrow x < 1$ $-3 < x < 1$

$(I) \cup (II) \rightarrow D_f = [-3, +\infty)$

$$f(x) = x^2 - 5x$$

$$f\left(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}\right) = f\left(\frac{\sqrt{2}}{2} + 1\right) \rightarrow 2\left(1 + \frac{\sqrt{2}}{2}\right) - 5\left(1 + \frac{\sqrt{2}}{2}\right) = \boxed{-\frac{1}{2}}$$

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$$f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a$$

$$D_f = \mathbb{R} - \{1\}$$

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$$f(x) = x^2 - 5x = -5$$

$$x^2 - 5x + 5 = 0$$

$$(x-5)^2 = 0 \rightarrow \boxed{x=5} \rightarrow g(a) = 5 \rightarrow \sqrt{x^2 - 1} = 5$$

$$x^2 - 1 = 25$$

$$\boxed{x = \pm \sqrt{26}}$$

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$$f(x+y) = (x-y)(x-y)$$

$$f(a) = (a-5)(a-5) \rightarrow f(5) = -5x - 1 = 5$$

$$f(5) = -5x - 1 = 5x$$

$$\boxed{15x}$$

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$$\cos x = t \rightarrow t^2 + \Delta t = \frac{1}{2} \Delta \rightarrow t^2 + \Delta t - \frac{1}{2} \Delta = 0$$

$$t^2 + \Delta t - \Delta t = 0$$

$$(t + \frac{1}{2})(t - \frac{1}{2}) = 0$$

$$\begin{cases} t = \frac{-\Delta}{\Delta} = -\frac{1}{2} \\ t = \frac{\Delta}{\Delta} = \frac{1}{2} \end{cases}$$

GGC

$$\cos x = \frac{1}{2} \rightarrow \tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \cos^2 x}{\cos^2 x} = \frac{1 - \frac{1}{4}}{\frac{1}{4}} = \boxed{\frac{3}{4}}$$

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