

$$\sin \frac{\pi}{6} = \frac{\sqrt{3}}{2} \quad \cos \frac{\pi}{6} = \frac{1}{2} \quad \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4} + \frac{\sqrt{3}}{2}$$

$$-\frac{\pi}{6} = -\frac{\pi}{6} \rightarrow \frac{\sqrt{3}}{2} + \frac{1}{2}$$

$$\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{\sqrt{3}}{2} = \sqrt{3} + 1$$

$$\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{1}{2}$$

$$f(a) = \frac{a+1}{a-1} \rightarrow f\left(\frac{a+1}{a-1}\right) = a \rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a$$

$$D_f = \mathbb{R} - \{1\}$$

برای هر عددی امکان پذیر!

$$g(a) = \sqrt{a^2 - 1} \rightarrow f(\sqrt{a^2 - 1}) = -\frac{1}{2} \rightarrow f(x) = x^2 - \frac{1}{2}x$$

$$\Rightarrow a^2 - 1 - \frac{1}{2}(\sqrt{a^2 - 1}) = -\frac{1}{2} \rightarrow a^2 - 1 - \frac{1}{2}\sqrt{a^2 - 1} + \frac{1}{2} = 0 \rightarrow \sqrt{a^2 - 1} = t$$

$$\Rightarrow t^2 - \frac{1}{2}t + \frac{1}{2} = (t-1)(t-\frac{1}{2}) = 0 \rightarrow \sqrt{a^2 - 1} = t = 1 \text{ or } \frac{1}{2}$$

$$a^2 - 1 = 1 \rightarrow a^2 = 2 \rightarrow a = \pm\sqrt{2}$$

$$f(x+3) = x^2 - 3x + 6 \Rightarrow (x-3)(x-2) \Rightarrow t = x+3, x = t-3$$

$$f(x) = (x-3-3)(x-3-2) = (x-6)(x-5) = x^2 - 11x + 30$$

$$f(-3) = 9 + 33 + 30 = 72 / f(4) = 16 - 44 + 30 = 2 \Rightarrow \sqrt{72} \times 2 = 144$$

$$\cos x = t \Rightarrow \frac{1}{2}t^2 + \frac{1}{2}t = \frac{1}{2} \rightarrow 1t^2 + 1t - 1 = 0$$

$$\Rightarrow t^2 + 1t - 1 = 0 \rightarrow (t+1)(t-1) = 0 \rightarrow t = -1 \text{ or } 1$$

$$\Rightarrow \cos x = \frac{1}{2} \rightarrow \sin x = \pm \frac{\sqrt{3}}{2} \Rightarrow \tan x = \pm \sqrt{3}$$

آترسیا آفاصله

تکلیف

$$y = \frac{1}{x+1} - \frac{2}{x-1} = 0$$

$$\frac{1}{x-1} + \frac{1}{x+1} = 2$$

$$\frac{x+1 + x-1}{(x-1)(x+1)} = 2$$

$$2x = 2 \Rightarrow x = 1$$

$$D_f = \mathbb{R} - \left\{ -\frac{1}{2}, -1, 1 \right\}$$

(الف) $y = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \geq 0$ (مشتق)

$$\frac{x^2 - 2x + 1 - x^2}{x(x-1)} = \frac{-2x+1}{x(x-1)} \geq 0$$

$$D_f = (-\infty, 0) \cup \left[\frac{1}{2}, 1\right)$$

(ب) $f(x) = \sqrt[3]{x-1} + \sqrt{y+1} = 3$

$$\sqrt[3]{x-1} - 3 = -\sqrt{y+1} \geq 0 \Rightarrow 0 \geq \sqrt{y+1}$$

(الف) $y = \sqrt{\left(\left(\frac{1}{x}\right)^x - 9\right) \left(3x - x^x\right)} \geq 0$

$$D_f = (-\infty, -2] \cup \left[\frac{5}{2}, +\infty\right)$$

langgo

$$\sqrt[3]{x-1} - 3 \leq 0 \Rightarrow \sqrt[3]{x-1} \leq 3$$

$$x-1 \leq 27 \Rightarrow x \leq 28$$

$$x-1 \geq 0 \Rightarrow x \geq 1$$

$$D_f = [1, 28]$$

(ج) $y = \frac{\log(x^x - x - 2)}{x^x - 1 + 1} > 0$

$$x^x - 1 > 0 \Rightarrow x^x > 1 \Rightarrow x > 1$$

$$D_f = (-\infty, -1) \cup (2, +\infty)$$

~~scribbles~~

$x^x - 1 \geq 0 \Rightarrow x^x \geq 1 \Rightarrow x \leq -1$ (ب) (د) $\Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

(هـ) $y = \sqrt{-x^x + ax + 3} \geq 0$

$$\Delta = \frac{1}{4} + 12 = \frac{49}{4}$$

$$x = \frac{-1 \pm 7}{-2} \Rightarrow x = -2 \text{ or } x = 3$$

$$a = -\frac{1}{2}$$

$$a + b = \frac{3}{2} - \frac{1}{2} = 1$$

(د) $f(x) - x \geq 0 \Rightarrow f(x) \geq x$

$$3x - 2 \geq x \Rightarrow x \geq 1$$

$$2x + 3 \geq x \Rightarrow x \geq -3 \wedge x < 1$$

دامنه $g(x) = [-3, 1) \cup [1, +\infty)$

$$[-3, +\infty)$$

$[1, +\infty)$