

$$b) f(x) = \frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{1}{x-1} + \frac{1}{x+1}} \rightarrow x+1 \neq 0 \rightarrow x \neq -1, x \neq 0, \frac{1}{x-1} + \frac{1}{x+1} \neq 0 \rightarrow \frac{x(x+1) + (x-1)}{(x+1)(x-1)} \neq 0 \rightarrow \frac{x^2 + 2x - 1}{(x+1)(x-1)} \neq 0 \rightarrow x \neq \frac{-2 \pm \sqrt{4 + 4}}{2} = -1, 1$$

$$\frac{x^2 + 2x - 1}{(x+1)(x-1)} = 0 \rightarrow x = \frac{-2 \pm \sqrt{4 + 4}}{2} = -1, 1$$

$$\rightarrow D_f = \mathbb{R} - \{-1, 1\}$$

$$\begin{aligned} \text{ب.) } \sqrt{x-1} + \sqrt{y+1} = v &\rightarrow x-1 \geq 0 \rightarrow \underbrace{x \geq 1}_{(I)}, y+1 \geq 0 \rightarrow \underbrace{y \geq -1}_{(II)}, \sqrt{y+1} = v - \sqrt{x-1} \rightarrow v - \sqrt{x-1} \geq 0 \\ &\rightarrow \sqrt{x-1} \leq v \rightarrow x-1 \leq 1 \rightarrow \underbrace{x \leq 1}_{(I)} \xrightarrow{I \cap II} \mathcal{D}_f = [1, 1] \end{aligned}$$

$$f(x) = \frac{\log_{1/2}(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \rightarrow x^2 - x - 2 > 0 \rightarrow (x - 2)(x + 1) > 0 \rightarrow \underbrace{-1 \quad 2}_{+\quad - \quad +} \quad (I)$$

$$\rightarrow x^2 - 1 > 0 \rightarrow x^2 > 1 \rightarrow x \geq 1 \vee x \leq -1 \quad (II) \quad \text{IN II} \cap \text{III} \rightarrow \mathcal{D}f = (-\infty, -1) \cup (2, +\infty)$$

$$\rightarrow \sqrt{x^2 - 1} + 1 \neq 0 \rightarrow \mathcal{R} \quad (III)$$

$$f(x) = \sqrt{r+ax-x^2} \rightarrow r+ax-x^2 \geq 0 \rightarrow x^2-ax-r \leq 0$$

$$D_f = [-r, b] \rightarrow \varepsilon' + ra - r = 0 \rightarrow a = -\frac{1}{r}$$

$$\rightarrow a = -r + b \rightarrow -\frac{1}{r} + r = b \rightarrow b = \frac{r-1}{r}$$

$$f(x) = \begin{cases} 3x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases} \quad \begin{aligned} & 3x-2 \geq x \rightarrow 2x \geq 2 \rightarrow x \geq 1 \rightarrow [1, +\infty) \text{ (I)} \\ & 2x+3 \geq x \rightarrow x \geq -3 \rightarrow (-\infty, 1) \cap [-3, +\infty) \rightarrow [-3, 1) \text{ (II)} \end{aligned}$$

$$g(x) = \sqrt{f(x)-x} \rightarrow f(x)-x \geq 0 \rightarrow f(x) \geq x \quad \xrightarrow{\text{I} \cup \text{II}} \cancel{g} = [-3, +\infty)$$

$$f(x) = \gamma x^2 - \varepsilon x$$

$$f\left(\sin \frac{\pi}{\gamma} + \cos^2 \pi\right) \rightarrow \sin \frac{\pi}{\gamma} = \frac{\sqrt{\gamma}}{\gamma}, \cos^2 \pi = 1 \rightarrow f\left(\frac{\sqrt{\gamma}}{\gamma} + 1\right)$$

$$\gamma \left(\frac{\sqrt{\gamma}}{\gamma} + 1\right)^2 - \varepsilon \left(\frac{\sqrt{\gamma}}{\gamma} + 1\right) = \gamma \left(\frac{\gamma}{\gamma} + \sqrt{\gamma} + 1\right) - \gamma \sqrt{\gamma} - \varepsilon = \frac{\gamma}{\gamma} + \cancel{\gamma \sqrt{\gamma}} + \gamma - \cancel{\gamma \sqrt{\gamma}} - \varepsilon = \frac{\gamma}{\gamma} - \varepsilon$$

$$f(x) = \frac{x+1}{x-1} \rightarrow f(f(a)) = a \rightarrow f(a) = \frac{a+1}{a-1} \rightarrow f\left(\frac{a+1}{a-1}\right) = a \rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a \quad (V)$$

$$\rightarrow a = \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{2a}{1} \rightarrow a \in \mathbb{R} - \{1\}$$

$$* x-1 \neq 0 \rightarrow x \neq 1 \Rightarrow a \neq 1$$

$$f(x) = x^2 - \varepsilon x \quad g_2(x) = \sqrt{x^2 - 1} \quad (A)$$

$$f(g_2(a)) = -\varepsilon \rightarrow f(t) = -\varepsilon \rightarrow t^2 - \varepsilon t = -\varepsilon \rightarrow t^2 - \varepsilon t + \varepsilon = 0 \rightarrow (t - \varepsilon)^2 = 0 \rightarrow t = \varepsilon$$

$$\rightarrow g_2(a) = \varepsilon \rightarrow \sqrt{a^2 - 1} = \varepsilon \rightarrow a^2 - 1 = \varepsilon \rightarrow a^2 = \Delta \rightarrow a = \pm \sqrt{\Delta}$$

$$f(x + \varepsilon) = x^2 - \Delta x + \varepsilon \quad (9)$$

$$\frac{f(\varepsilon) \times f(-\varepsilon)}{f(1+\varepsilon) \times f(-1+\varepsilon)} = (1 - \Delta + \varepsilon) \times (\varepsilon^2 + \varepsilon + \varepsilon) = \varepsilon \times \varepsilon^2 = \varepsilon^3$$

$$f(\varepsilon \cos x + \Delta \cos x) = \tan^2 x \quad (10)$$

$$f\left(\frac{1}{\varepsilon}, \Delta\right) = ?$$

$$\rightarrow \varepsilon \cos^2 x + \Delta \cos x = \frac{1}{\varepsilon}, \Delta \rightarrow \varepsilon \cos^2 x + \Delta \cos x - \frac{1}{\varepsilon} = 0$$

$$\rightarrow 1 \cos^2 x + 10 \cos x - 1 = 0 \rightarrow \cos^2 x + 10 \cos x - \Delta \varepsilon = 0$$

$$\rightarrow (\cos x + \varepsilon)(\cos x - \varepsilon) = 0$$

$$\rightarrow \cos x = \frac{1}{\varepsilon}, \frac{-10}{2} = -5$$

$$-1 \leq \cos x \leq 1 \leftarrow \text{مطلوب}$$

$$\rightarrow \cos x = \frac{1}{\varepsilon} \rightarrow \tan^2 x = \left(\frac{\sin x}{\cos x}\right)^2$$

$$\rightarrow \tan^2 x = \left(\frac{\frac{\sqrt{1-\frac{1}{\varepsilon^2}}}{\frac{1}{\varepsilon}}}{\frac{1}{\varepsilon}}\right)^2 = \frac{1-\frac{1}{\varepsilon^2}}{\frac{1}{\varepsilon^2}} = \varepsilon^2 - 1$$

$$\rightarrow f\left(\frac{1}{\varepsilon}, \Delta\right) = \varepsilon^2 - 1$$