

(1)

$$f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{-2x + 1}{x(x-1)} \leq 0 \rightarrow \frac{1}{x} - \frac{2}{x-1} \leq 0$$

$$\rightarrow D_f = (-\infty, 0) \cup \left[\frac{1}{2}, 1\right)$$

$$b) f(x) = \frac{\frac{1}{x+1} - \frac{y}{x}}{\frac{1}{x-1} + \frac{1}{x+2}} \rightarrow x+1 \neq 0 \rightarrow x \neq -1, x \neq 0, \frac{y}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow \frac{y(x+2) + (x-1)}{(x+2)(x-1)} \neq 0$$

$$\frac{yx + 2y + x - 1}{(x+2)(x-1)} \neq 0 \rightarrow x \neq \frac{-2y-1}{y}, -2, 1 \rightarrow D_f = \mathbb{R} - \left\{-2, -\frac{2y-1}{y}, -1, 0, 1\right\}$$

(2)

$$f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^x - 9\right)(32 - x^x)} \rightarrow \left(\left(\frac{1}{x}\right)^x - 9\right)(32 - x^x) \geq 0 \rightarrow (x^{-x} - 9)(32 - x^x) \geq 0 \rightarrow x^{-x} - 9 = 0 \rightarrow x = -2$$

$$\rightarrow 32 - x^x = 0 \rightarrow x = \frac{2}{y} \rightarrow \frac{-2}{\frac{2}{y}} \rightarrow D_f = (-\infty, -2] \cup \left[\frac{2}{y}, +\infty\right)$$

$$b) \sqrt{x-1} + \sqrt{y+1} = 3 \rightarrow x-1 \geq 0 \rightarrow x \geq 1, y+1 \geq 0 \rightarrow y \geq -1, \sqrt{y+1} = 3 - \sqrt{x-1} \rightarrow 3 - \sqrt{x-1} \geq 0$$

$$\rightarrow \sqrt{x-1} \leq 3 \rightarrow x-1 \leq 9 \rightarrow x \leq 10 \xrightarrow{I \cap II} D_f = [1, 10]$$

(3)

$$f(x) = \frac{\log_{1/2}(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \rightarrow x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow \frac{-1}{x} - \frac{2}{x+1} > 0 \quad (I)$$

$$\rightarrow x^2 - 1 > 0 \rightarrow x^2 > 1 \rightarrow x > 1 \vee x < -1 \quad (II) \xrightarrow{I \cap II \cap III} D_f = (-\infty, -1) \cup (2, +\infty)$$

$$\rightarrow \sqrt{x^2 - 1} + 1 \neq 0 \rightarrow \mathbb{R} \quad (III)$$

(4)

$$f(x) = \sqrt{3 + ax - x^2} \rightarrow 3 + ax - x^2 \geq 0 \rightarrow x^2 - ax - 3 \leq 0$$

$$D_f = [-r, b] \rightarrow \varepsilon + 2a - 3 = 0 \rightarrow a = -\frac{1}{2} \quad a + b = -\frac{1}{2} + \frac{3}{2} = \frac{2}{2} = 1$$

$$\rightarrow a = -2 + b \rightarrow -\frac{1}{2} + 2 = b \rightarrow b = \frac{3}{2}$$

(5)

$$f(x) = \begin{cases} 3x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases} \rightarrow \begin{cases} 3x-2 \geq x \rightarrow 2x \geq 2 \rightarrow x \geq 1 \rightarrow [1, +\infty) \quad (I) \\ 2x+3 \geq x \rightarrow x \geq -3 \rightarrow (-\infty, 1) \cap [-3, +\infty) \rightarrow [-3, 1) \quad (II) \end{cases}$$

$$g(x) = \sqrt{f(x) - x} \rightarrow f(x) - x \geq 0 \rightarrow f(x) \geq x \xrightarrow{I \cup II} D_g = [-3, +\infty)$$

(6)

$$f(x) = 2x^2 - 2x$$

$$f\left(\sin \frac{\pi}{3} + \cos 2x\right) \rightarrow \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}, \cos 2x = 1 \rightarrow f\left(\frac{\sqrt{3}}{2} + 1\right)$$

$$2\left(\frac{\sqrt{3}}{2} + 1\right)^2 - 2\left(\frac{\sqrt{3}}{2} + 1\right) = 2\left(\frac{3}{4} + \sqrt{3} + 1\right) - 2\left(\frac{\sqrt{3}}{2} + 1\right) = \frac{3}{2} + 2\sqrt{3} + 2 - \sqrt{3} - 2 = \frac{1}{2}$$

$$f(x) = \frac{x+1}{x-1} \rightarrow f(f(a)) = a \rightarrow f(a) = \frac{a+1}{a-1} \rightarrow f\left(\frac{a+1}{a-1}\right) = a \rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a \quad (V)$$

$$\rightarrow a = \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{2a}{1} \rightarrow a \in \mathbb{R} - \{1\}$$

$$* x-1 \neq 0 \rightarrow x \neq 1 \Rightarrow a \neq 1$$

$$f(x) = x^2 - \varepsilon x \quad g(x) = \sqrt{x^2 - 1} \quad (A)$$

$$f(g(a)) = -\varepsilon \rightarrow f(t) = -\varepsilon \rightarrow t^2 - \varepsilon t = -\varepsilon \rightarrow t^2 - \varepsilon t + \varepsilon = 0 \rightarrow (t - \varepsilon)^2 = 0 \rightarrow t = \varepsilon$$

$$\rightarrow g(a) = \varepsilon \rightarrow \sqrt{a^2 - 1} = \varepsilon \rightarrow a^2 - 1 = \varepsilon \rightarrow a^2 = \varepsilon + 1 \rightarrow a = \pm \sqrt{\varepsilon + 1}$$

$$f(x + \varepsilon) = x^2 - \varepsilon x + \varepsilon \quad (9)$$

$$\frac{f(\varepsilon) \times f(-\varepsilon)}{f(1+\varepsilon) \times f(-1-\varepsilon)} = (1 - \varepsilon + \varepsilon) \times (\varepsilon^2 + \varepsilon + \varepsilon) = \varepsilon \times \varepsilon = \varepsilon^2$$

$$f(\varepsilon \cos x + \Delta \cos x) = \tan^2 x \quad (10)$$

$$f(\varepsilon, \Delta) = ?$$

$$\rightarrow \varepsilon \cos^2 x + \Delta \cos x = \varepsilon, \Delta \rightarrow \varepsilon \cos^2 x + \Delta \cos x - \frac{\varepsilon}{\cos x} = 0$$

$$\rightarrow 1 \cos^2 x + 10 \cos x - \varepsilon = 0 \rightarrow \cos^2 x + 10 \cos x - \varepsilon = 0$$

$$\rightarrow (\cos x + \varepsilon)(\cos x - \varepsilon) = 0$$

$$\rightarrow \cos x = \frac{1}{\varepsilon}, \frac{-10}{\varepsilon}$$

$$-1 \leq \cos x \leq 1 \Rightarrow \varepsilon \geq 10$$

$$\rightarrow \cos x = \frac{1}{\varepsilon} \rightarrow \tan^2 x = \left(\frac{\sin x}{\cos x}\right)^2$$

$$\rightarrow \tan^2 x = \left(\frac{\frac{\sqrt{1-\frac{1}{\varepsilon^2}}}{\frac{1}{\varepsilon}}}{\frac{1}{\varepsilon}}\right)^2 = \varepsilon$$

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$$\rightarrow f(\varepsilon, \Delta) = \varepsilon$$