

$$f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \rightarrow \frac{(x-1)^{\frac{p}{2}} - x^{\frac{p}{2}}}{x(x-1)} \geq 0$$

$$\begin{array}{c|c|c} 0 & \frac{1}{x} & 1 \\ \hline + & - & + \end{array}$$

$$\frac{x^{\frac{p}{2}} + 1 - x^{\frac{p}{2}} - x^{\frac{p}{2}}}{x(x-1)} = \sqrt{\frac{1-x^{\frac{p}{2}}}{x(x-1)}} \xrightarrow{II} 1-x^{\frac{p}{2}} \geq 0 \rightarrow \frac{x^{\frac{p}{2}}}{x} \geq 1 \xrightarrow{III} x^{\frac{p}{2}} \geq x \rightarrow x^{\frac{p}{2}} \geq x^1$$

$$D_f = (-\infty, 0) \cup \left[\frac{1}{p}, 1 \right)$$

$$f(x) = \frac{\frac{1}{x+1} - \frac{x}{x-1}}{\frac{x}{x-1} + \frac{1}{x+1}} \rightarrow (I) x \neq -1 \text{ و } x \neq -\frac{1}{p}$$

$$f(x) + 0 \neq 0 \rightarrow x \neq -\frac{1}{p}$$

$$(II) \frac{x}{x-1} + \frac{1}{x+1} \neq 0 \rightarrow \frac{x(x+1) + 1(x-1)}{(x-1)(x+1)} = \frac{x^2 + x + x - 1}{(x-1)(x+1)} = \frac{x^2 + 2x - 1}{(x-1)(x+1)}$$

$$D_f = \mathbb{R} - \left\{ -1, -\frac{1}{p} \right\}$$

$$x \neq 1 \text{ و } x \neq -\frac{1}{p}$$

$$f(x) = \sqrt{\left[\left(\frac{1}{x} \right)^n - 9 \right] (x^p - x^q)}$$

$$(I) \left(\frac{1}{x} \right)^n - 9 \geq 0 \rightarrow x^{-n} \geq 9 \rightarrow x \leq -\frac{1}{9} \text{ (باید عبارت را دقیق حساب می کردی!)} \quad \frac{-2}{+b} - \frac{a}{+} + \frac{c}{+}$$

$$(II) x^p - x^q \geq 0 \rightarrow x^p \geq x^q \rightarrow \infty \geq x^q - x^p \rightarrow \frac{0}{p} \geq x \rightarrow D_f = (-\infty, -\frac{1}{9}] \cup [\frac{0}{p}, +\infty)$$

$$(I) \cap (II) \rightarrow D_f = (-\infty, -\frac{1}{9}]$$

$$c) \sqrt{x-1} + \sqrt{y+1} = x^{\frac{p}{2}} \rightarrow \sqrt{y+1} = x^{\frac{p}{2}} - \sqrt{x-1} \rightarrow \sqrt{y+1} \leq x^{\frac{p}{2}} \rightarrow x-1 \leq 1 \rightarrow x \leq 2$$

$$(I) x-1 \geq 0 \rightarrow x \geq 1 \rightarrow [1, +\infty)$$

$$D_f = [1, 2]$$

$$f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \rightarrow I) x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow x > 2 \text{ و } x < -1$$

$$\begin{array}{c|c|c} -1 & 2 & \\ \hline + & - & + \end{array}$$

$$[1, +\infty) \cup (-\infty, -1]$$

$$II) x^2 - 1 \geq 0 \rightarrow x \geq 1 \text{ و } x \leq -1$$

$$III) \sqrt{x^2 - 1} + 1 \neq 0 \rightarrow \sqrt{x^2 - 1} \neq -1 \rightarrow \mathbb{R}$$

$$I \cap II \cap III \rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$$

$$f(x) = \sqrt{x^p + ax - x^q} \rightarrow x = -2 \rightarrow y = 0 \rightarrow x^p - x^q - 1 = 0 \rightarrow -1 - 2^p = 0 \rightarrow a = \frac{1}{p}$$

$$[-2, b] \quad a+b=2 \quad (x^p - \frac{1}{p}x - x^q = 0) \rightarrow (x - \frac{p}{p})(x + \frac{p}{p}) \rightarrow \frac{0}{p} = -2 \rightarrow \frac{p}{p}$$

$$b = \frac{p}{p} / a = -\frac{1}{p} \rightarrow a+b=1$$

$$f(x) = \begin{cases} px - p & x \geq 1 \\ px + p & x < 1 \end{cases} \quad g(x) = \sqrt{px - 1}$$

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$$I \rightarrow x \geq 1 \rightarrow f(x) - x = px - p - x = px - p \rightarrow px - p \geq 0 \rightarrow x \geq 1 \rightarrow \mathbb{R}_{\geq 1}$$

$$II \rightarrow x < 1 \rightarrow f(x) - x = px + p - x = x + p \rightarrow x + p \geq 0 \rightarrow x \geq -p$$

$$I \cup II = [-p, +\infty)$$

$$f(x) = px^p - px \rightarrow px(x - 1) \rightarrow p \left(1 + \frac{\sqrt{p}}{p}\right) \left(-1 + \frac{\sqrt{p}}{p}\right) \rightarrow p \left(\frac{p}{p} - 1\right) = -1$$

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$$f(\sin \frac{\pi}{p} + \cos \frac{\pi}{p}) = \sin \frac{\pi}{p} = \frac{\sqrt{p}}{p} \rightarrow 1 + \frac{\sqrt{p}}{p}$$

$$f(x) = \frac{x+1}{x-1} / f(f(x)) = x$$

$$f(x) = \frac{x+1}{x-1} \rightarrow f(f(x)) = \frac{f(x)+1}{f(x)-1} = \frac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1} \rightarrow \frac{\frac{x+1+x-1}{x-1}}{\frac{x+1-x+1}{x-1}} = \frac{2x}{2} = x$$

$$f(f(x)) = \frac{px}{x-1} = \frac{px}{x-1} \times \frac{x-1}{p} = x$$

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$$0 \rightarrow \alpha - 1 \neq 0 \rightarrow \alpha \neq 1 \rightarrow \alpha \in \mathbb{R} \setminus \{1\}$$

$$f(x) = x^p - px / g(x) = \sqrt{x^p - 1} / f(g(x)) = -p / \alpha = p$$

$$g(x) = \sqrt{x^p - 1}$$

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$$f(x) = (x^p - px) / g(x) \rightarrow f(g(x)) = (g(x))^p - pg(x) \rightarrow g(x)^p - pg(x) = 0$$

$$g(x) = \sqrt{x^p - 1} \rightarrow p = \sqrt{x^p - 1} \rightarrow x^p - 1 = p^2 \rightarrow x = \pm \sqrt[p]{p^2 + 1}$$

$$f(x+p) = x^p - px + p$$

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$$f_c(f) \rightarrow x + p = p \rightarrow x = 1$$

$$f_c(f) = 1^p - p \cdot 1 + p = p$$

$$f_c(-p) \rightarrow x + p = -p \rightarrow x = -1 \rightarrow f_c(f) f_c(-p) = p \cdot p = p^2$$

$$f(p \cos^p x + q \cos x) = \tan^p x / f(p, q) = ?$$

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$$7 \cos^2 x + 6 \cos x = 4 \quad | \quad x^2 + 1 \cos^2 x + 1 \cos x = 1 \rightarrow 1 \cos^2 x + 1 \cos x - 1 = 0 \rightarrow z = \cos x$$

$$1z^2 + 1z - 1 = 0 \rightarrow z^2 + 1z - 1 = 0 \rightarrow \Delta = 1 + 4 = 5 \rightarrow z = \frac{-1 \pm \sqrt{5}}{2} \rightarrow z = \frac{1}{4}, -\frac{1}{4}$$

$$z = \cos x \rightarrow \frac{1}{4} \in [0, 1] \quad \checkmark$$

$$\rightarrow \frac{1}{4} \quad \checkmark \quad \sin^2 x = 1 - \cos^2 x$$

$$\sin^2 x = \frac{1 - \cos^2 x}{\cos^2 x} \rightarrow \frac{1 - \frac{1}{4}}{\frac{1}{4}} = 3 \rightarrow \sin^2 x = 3 \quad \checkmark$$

(2-الف) $\left(\left(\frac{1}{x}\right)^n - 9\right)(x^2 - 2) \geq 0 \rightarrow$

$\left(\frac{1}{x}\right)^n = 9 \rightarrow n = -2 \quad x^2 = 2 \rightarrow x = \pm \sqrt{2}$

$\frac{-2}{+} \quad \frac{2}{-} \rightarrow D_f = (-\infty, -\sqrt{2}] \cup [\sqrt{2}, +\infty)$

(2-ب) $\sqrt[n]{x-1} + \sqrt[n]{y+1} = 2$, $\sqrt[n]{y+1} = 2 - \sqrt[n]{x-1} \rightarrow 2 - \sqrt[n]{x-1} \geq 0 \rightarrow \sqrt[n]{x-1} \leq 2 \rightarrow x \leq 17$

$x-1 \geq 0 \rightarrow x \geq 1 \quad I$

$I, II \rightarrow 1 \leq x \leq 17 \rightarrow D_f = [1, 17]$