

$$f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \rightarrow \frac{(x-1)^p - x^p}{x(x-1)} \geq 0$$

$$\begin{array}{c} 0 & \frac{1}{p} & 1 \\ + & \frac{1}{p} & - \\ - & \frac{1}{p} & + \end{array}$$

$$\frac{x^p + 1 - px}{x(x-1)} = \sqrt{\frac{1-px}{x(x-1)}} \xrightarrow{\text{II}} 1-px \geq 0 \rightarrow \frac{1}{p} \geq x \xrightarrow{\text{III}} x(x-1) > 0 \rightarrow \frac{1}{p} \geq 1$$

$$D_f = (-\infty, 0) \cup \left[\frac{1}{p}, 1\right)$$

$$f(x) = \frac{\frac{1}{x+1} - \frac{x}{x-1}}{\frac{x}{x-1} + \frac{1}{x+p}} \rightarrow (x \neq -1, 0, 1, -p)$$

$$f(x) + 0 \neq 0 \rightarrow x \neq -\frac{1}{p}$$

$$\text{II}, \frac{x}{x-1} + \frac{1}{x+p} \neq 0 \rightarrow \frac{p(x+p) + 1(x-1)}{(x-1)(x+p)} = \frac{px + p^2 + x - 1}{(x-1)(x+p)} = \frac{f(x) + 0}{(x-1)(x+p)}$$

$$x \neq 1, -p$$

$$D_f = \mathbb{R} - \left\{ -1, -\frac{1}{p}, 0, 1 \right\}$$

$$f(x) = \sqrt{\left(\frac{1}{x}\right)^p - 1} \sqrt{px - 1}$$

$$\text{I}, \left(\frac{1}{x}\right)^p - 1 \geq 0 \rightarrow x^{-p} \geq 1 \rightarrow x \leq -1$$

$$\text{II}, px - 1 \geq 0 \rightarrow x \geq \frac{1}{p} \rightarrow \frac{1}{p} \geq x$$

$$\text{I} \cap \text{II} \rightarrow \frac{1}{p} \geq x \leq -1 \rightarrow D_f = (-\infty, -1]$$

$$\text{III}, \sqrt{x-1} + \sqrt{y+1} = \frac{1}{p}$$

$$\text{I}, x-1 \geq 0 \rightarrow x \geq 1 \rightarrow [1, +\infty)$$

$$f(x) = \frac{\log(x^p - x - p)}{\sqrt{x^p - 1} + 1} \rightarrow I_1 = x^p - x - p > 0 \rightarrow (x-p)(x+1) > 0 \rightarrow x > p \text{ or } x < -1$$

$$[1, +\infty) \cup (-\infty, -1]$$

$$\text{II}, x^p - 1 \geq 0 \rightarrow x \geq 1 \text{ or } x \leq -1$$

$$\text{III}, \sqrt{x^p - 1} + 1 \neq 0 \rightarrow \sqrt{x^p - 1} \neq -1 \rightarrow \mathbb{R}$$

$$I \cap II \cap III \rightarrow D_f = (-\infty, -1) \cup (p, +\infty)$$

$$f(x) = \sqrt{p + ax - x^p} \rightarrow x = -p \rightarrow y = 0 \rightarrow p - pa - p \rightarrow -1 - pa = 0 \rightarrow a = -\frac{1}{p}$$

$$[-p, b] \quad a+b=1 \quad \left(p - \frac{1}{p}x - x^p = 0\right) \rightarrow \left(x - \frac{p}{p}\right)(x+p) \rightarrow \frac{1}{p} = -p \rightarrow \frac{1}{p}$$

$$b = \frac{p}{p} / a = -\frac{1}{p} \rightarrow a+b=1$$

$$f(x) = \begin{cases} px - p & x \geq 1 \\ px + p & x < 1 \end{cases} \quad g(x) = \sqrt{p(x) - x}$$

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$$I \rightarrow x \geq 1 \rightarrow f(x) - x = px - p - x = px - p \rightarrow px - p \geq 0 \rightarrow x \geq 1 \rightarrow \mathbb{I}_1 + \infty$$

$$II \rightarrow x < 1 \rightarrow f(x) - x = px + p - x = x + p \rightarrow x + p \geq 0 \rightarrow x \geq -p$$

$$I \cup II = \begin{array}{c} \bullet \text{---} \circ \\ \text{---} \text{---} \text{---} \end{array} \rightarrow D_f = [-p, +\infty)$$

$$f(x) = px^p - x \rightarrow p(x(x-p)) \rightarrow p \left(1 + \frac{\sqrt{p}}{p}\right) \left(-1 + \frac{\sqrt{p}}{p}\right) \rightarrow p \left(\frac{p}{p} - 1\right) = \frac{-1}{p}$$

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$$f(\sin \frac{\pi}{p} + \cos \frac{\pi}{p}) = \sin \frac{\pi}{p} = \frac{\sqrt{p}}{p} \rightarrow 1 + \frac{\sqrt{p}}{p}$$

$$f(x) = \frac{x+1}{x-1} / f(f(x)) = x$$

$$f(x) = \frac{x+1}{x-1} \rightarrow f(f(x)) = \frac{f(x)+1}{f(x)-1} = \frac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1} \rightarrow \frac{\frac{x+1+x-1}{x-1}}{\frac{x+1-x+1}{x-1}} = \frac{2x}{2} = x$$

$$f(f(x)) = \frac{px}{x-1} = \frac{px}{x-1} \times \frac{x-1}{p} = x$$

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$$0 \rightarrow \lim_{x \rightarrow 0} \frac{1}{x} \rightarrow x-1 \neq 0 \rightarrow x \neq 1 \rightarrow x \in \mathbb{R} \setminus \{1\}$$

$$f(x) = x^p - x / g(x) = \sqrt{x^p - 1} / f(g(x)) = -x / x = -1$$

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$$f(x) = (x^p - x) / g(x) \rightarrow f(g(x)) = (g(x))^p - x / g(x) \rightarrow \frac{(g(x))^p - x}{g(x)} = \frac{(x^p - 1) - x}{\sqrt{x^p - 1}} = \frac{x^p - x - 1}{\sqrt{x^p - 1}}$$

$$g(x) = \sqrt{x^p - 1} \rightarrow x = \sqrt{x^p - 1} \rightarrow x^p - 1 = x^2 \rightarrow x = \pm \sqrt{x^p - 1}$$

$$f(x+p) = x^p - px + p$$

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$$f(f) \rightarrow x + p = p \rightarrow x = 1$$

$$f(f) = 1^p - p + p = p$$

$$f(-p) \rightarrow x + p = -p \rightarrow x = -p \rightarrow f(f) f(-p) = 1^p - p = 1 - p$$

$$f(-p) = p^p - p + p = p^p$$

$$f(p \cos x + q \cos x) = \tan^p x / f(p, q) = ?$$

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$$F \cos^2 \alpha + G \cos \alpha = \frac{1}{2} \rightarrow 1 \cos^2 \alpha + 1 \cos \alpha = \frac{1}{2} \rightarrow 1 \cos^2 \alpha + 1 \cos \alpha - \frac{1}{2} = 0 \rightarrow z = \cos \alpha$$

$$1z^2 + 1z - \frac{1}{2} = 0 \rightarrow z^2 + 1z - \frac{1}{2} \rightarrow (z - \frac{1}{2}), (z + 1\frac{1}{2}) \rightarrow \frac{-1F}{1} \pm \frac{F}{1} \rightarrow z = \frac{1}{2}, -\frac{1}{2}$$

$$z = \cos \alpha \rightarrow \frac{1}{2} \in \mathbb{C} \cap \mathbb{R}$$

$$\rightarrow \frac{1}{2} \quad \text{OK} \quad \checkmark$$

$$z \sin^2 \alpha = \frac{\sin^2 \alpha}{\cos^2 \alpha} \rightarrow \sin^2 \alpha = 1 - \cos^2 \alpha$$

$$z \sin^2 \alpha = \frac{1 - \cos^2 \alpha}{\cos^2 \alpha} \rightarrow \frac{1 - \frac{1}{4}}{\frac{1}{4}} = 3 \rightarrow f(1/2) = 3$$
