

۲. آزمون عالی فونته

$$f(x) = \begin{cases} (a+1)(x+2) & ; x > 1 \\ 2a+2x & ; x \leq 1 \end{cases}$$

$$2f(2) = f(-2) + a \rightarrow a = ? \quad f(a+1) = (3a-4) + a$$

$$1fa + 1f = fa - f$$

$$1 \cdot a = -1 \quad a = \frac{-1}{1} = -1$$

$$f(2) = (a+1)(2+2) = 4(a+1) \sim 2f(2) = 1f(a+1)$$

$$f(-2) = 2a + 2(-2) = 2a - 4$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$$

$$f(2-\sqrt{3}) + f(2+\sqrt{3}) = ? \rightarrow 2\sqrt{9} + 4$$

$$S \cdot \sqrt{S} = (2-\sqrt{3})(2+\sqrt{3}) = 1 \sim \sqrt{S} = \frac{1}{\sqrt{S}} \rightarrow f(S) + f(\sqrt{S}) = \sqrt{S} + \frac{1}{\sqrt{S}} + 2 + \frac{1}{\sqrt{S}} + 2$$

$$\frac{1}{\sqrt{S}} = \sqrt{S} \Rightarrow 2\sqrt{S} + 2\sqrt{S} + 4 = 2(\sqrt{S} + \sqrt{S}) + 4 = 4\sqrt{S} + 4$$

$$* (\sqrt{S} + \sqrt{S})^2 = S + S + 2\sqrt{S} \cdot \sqrt{S} = 2 + 2 + 2\sqrt{S} \cdot \sqrt{S} = 4 + 2\sqrt{S} \cdot \sqrt{S} = 4 + 2 \cdot 1 = 6 \rightarrow \sqrt{S} + \sqrt{S} = \sqrt{6} \quad \sqrt{S} \cdot \sqrt{S} = 1$$

$$f(x) = ?$$

$$2f(x) - 3f(-x) = f(x^2 - x)$$

$$\begin{cases} 2f(x) - 3f(-x) = f(x^2 - x) \\ 2f(x) - 3f(-x) = f(x^2 - x) \end{cases}$$

$$f(x) = -\frac{1}{2}x^2 - \frac{1}{2}x$$

$$-2f(x) = 2 \cdot x^2 + x$$

$$(x+1)f(x) - 3xf(x+2) = f(x^2 - mx + m - 1)$$

$$f(0) = ?$$

$$x=0 \rightarrow 2f(0) = 3m-1 \rightarrow f(0) = \frac{3m-1}{2}$$

$$x=-2 \rightarrow 9f(0) = 19+2m-1 \rightarrow f(0) = \frac{12+2m}{9}$$

$$\sim f(0) = \frac{3 \times \frac{12+2m}{9} - 1}{2} = \frac{\frac{12+2m}{3} - 1}{2} = \frac{\frac{12+2m-3}{3}}{2} = \frac{9+2m}{6}$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 - 12x + 12}{x}$$

$$f(-1) = ? \rightarrow f(-1) = -9$$

$$f(x) = ax + b$$

$$f(x) + f\left(\frac{1}{x}\right) = ax + b + \frac{a}{x} + b = \left(ax + \frac{a}{x} + 2b\right)$$

$$f(x) = ax + b$$

$$f\left(\frac{1}{x}\right) = \frac{a}{x} + b$$

$$\sim f(x) = 3x - 9 \sim f(-1) = 3(-1) - 9 = -12$$

$$\begin{cases} f(x) = (x-1)\sqrt{1-x} \\ g(x) = -\sqrt{(1-x)^3} \end{cases}$$

سین و این تابع

① می‌گیرن لانه $\begin{cases} f(x): 1-x \geq 0 \rightarrow x \leq 1 \\ g(x): 1-x \geq 0 \rightarrow x \leq 1 \end{cases}$

② می‌گیرن ضابطه $\begin{cases} f(x): (x-1)\sqrt{1-x} \\ g(x): -\sqrt{(1-x)^3} \end{cases}$ چون $x \leq 1$ $-(1-x)\sqrt{1-x} = (x-1)\sqrt{1-x}$

$$\begin{cases} f(x) = x\sqrt{x} \\ g(x) = \sqrt{x^3} \end{cases}$$

سین این تابع با هم مازند

① می‌گیرن لانه $\begin{cases} f(x): x \geq 0 \\ g(x): x \geq 0 \end{cases}$

② می‌گیرن ضابطه $\begin{cases} f(x): x\sqrt{x} \\ g(x): \sqrt{x^3} \end{cases} \Rightarrow |x|\sqrt{x} \xrightarrow{x \geq 0} x\sqrt{x}$

$$\begin{cases} f(x) = [\sqrt{x} - \lfloor \sqrt{x} \rfloor] \\ g(x) = \sqrt{x - |x|} \end{cases}$$

این تابع سین و هم لانه و هم ضابطه آن خارج است

① می‌گیرن لانه $f(x): [\sqrt{x} - \lfloor \sqrt{x} \rfloor]$ $g(x): \sqrt{x - |x|}$
 $\sqrt{x} - \lfloor \sqrt{x} \rfloor = \sqrt{x} - n$ $0 \leq \sqrt{x} - n < 1 \Rightarrow n \leq \sqrt{x} < n+1 \Rightarrow \lfloor \sqrt{x} \rfloor = n$
 $g(x): \sqrt{x - |x|} \Rightarrow x - |x| \geq 0 \Rightarrow x \geq 0$
 $g(x) = 0 \quad (x \geq 0)$

$$f = \{(1,2), (2,3), (3,1), (0,0)\}, g = \{(-1,1), (1,1), (2,2), (3,5), (0,2), (\sqrt{2},1)\}$$

الف) $\left(\frac{f+g}{f-g}\right)(1) \xrightarrow{x=1} \begin{cases} f(1)=2 \\ g(1)=1 \end{cases} \rightarrow \frac{f(1)+g(1)}{f(1)-g(1)} = \frac{2+1}{2-1} = 3$

ب) $\left(\frac{f+g}{f-g}\right)(2) \xrightarrow{x=2} \begin{cases} f(2)=3 \\ g(2)=2 \end{cases} \rightarrow \frac{f(2)+g(2)}{f(2)-g(2)} = \frac{3+2}{3-2} = 5$

ج) $f(0)g(-1) \rightarrow \begin{cases} f(0)=0 \\ g(-1)=1 \end{cases} \rightarrow f(0)g(-1) = 0$

$$f(x) = \{(2,1), (-1,4), (0,5), (-2,2), (3,-1)\}, g = \{(1,1), (0,4), (\frac{1}{2},2), (\frac{4}{3},-1)\}$$

الف) $g(x) = f(3x-1)$
 $2x-1=2 \rightarrow 2x=3 \rightarrow x=\frac{3}{2} \rightarrow g(\frac{3}{2})=f(2)=1$
 $-1x-1=-1 \rightarrow -x=0 \rightarrow x=0 \rightarrow g(0)=f(-1)=4$
 $0x-1=0 \rightarrow -x=1 \rightarrow x=-1 \rightarrow g(-1)=f(0)=5$
 $-2x-1=-2 \rightarrow -2x=-1 \rightarrow x=\frac{1}{2} \rightarrow g(\frac{1}{2})=f(-2)=2$
 $3x-1=-1 \rightarrow 3x=0 \rightarrow x=0 \rightarrow g(0)=f(3)=-1$

ب) $h(x) = f(|x|)$

ج) $\begin{cases} x=2 \rightarrow |x|=2 \rightarrow f(2)=1 \rightarrow h(2)=1 \\ x=-1 \rightarrow |x|=1 \rightarrow f(1)=4 \rightarrow h(-1)=4 \\ x=0 \rightarrow |x|=0 \rightarrow f(0)=5 \rightarrow h(0)=5 \\ x=-2 \rightarrow |x|=2 \rightarrow f(2)=1 \rightarrow h(-2)=1 \\ x=3 \rightarrow |x|=3 \rightarrow f(3)=-1 \rightarrow h(3)=-1 \end{cases}$

د) $h = \{(-3,-1), (-2,1), (0,5), (2,1), (3,-1)\}$