

Subject:

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اساتذہ کرام کی خدمت میں تحفہ
 Name.

$$f(x) = \begin{cases} (a+1)(x+2) : x > 1 & 2f(x) = f(-2) + a \rightarrow 2(a+1)(x) \\ 3a+2x : x \leq 1 & 2(a+1)(x) = (3a-2) + a \end{cases} \quad \text{(سوال 1)}$$

$$\rightarrow 2(a+1) = 3a-2 \rightarrow 2a+2 = 3a-2 \rightarrow 10a = -4 \rightarrow a = -\frac{4}{10} = -\frac{2}{5} = -0.4$$

جواب: -0.4

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$$

$$f(2-\sqrt{3}) + f(2+\sqrt{3}) = \left(\sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2\right) + \left(\sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2\right)$$

$$= \left(\sqrt{2} + \frac{1}{\sqrt{2}} + 2\right) + \left(\frac{1}{\sqrt{2}} + \sqrt{2} + 2\right)$$

$$\sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2 + \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2 \rightarrow 2\sqrt{2-\sqrt{3}} + \frac{2}{\sqrt{2-\sqrt{3}}} + 4$$

$$\rightarrow 2(\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}}) + 4 \rightarrow (\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}})^2 = 2 - \sqrt{3} + 2 + \sqrt{3} = 4$$

$$\rightarrow 2(\sqrt{4}) + 4 \rightarrow f(2-\sqrt{3}) + f(2+\sqrt{3}) = 4 + 2\sqrt{4}$$

$$2f(x) - 3f(-x) = 5x^2 - x$$

(سوال 3)

$$f(x) = ? \rightarrow -5x^2 - \frac{1}{5}x$$

$$\rightarrow 2f(-x) - 3f(-(-x)) = 5(-x)^2 - (-x) = 2f(-x) - 3f(x) = 5x^2 + x$$

$$\left. \begin{aligned} 2 \times (2f(x) - 3f(-x)) &= 5x^2 - x \\ 3 \times (2f(-x) - 3f(x)) &= 5x^2 + x \end{aligned} \right\} \rightarrow \begin{aligned} 4f(x) - 6f(-x) &= 5x^2 - x \\ 6f(-x) - 9f(x) &= 5x^2 + x \end{aligned}$$

$$-5f(x) = 20x^2 + x \rightarrow f(x) = \frac{20x^2 + x}{-5} \rightarrow f(x) = -4x^2 - \frac{1}{5}x$$

$$(n+2)f(n) - 3nf(n+2) = 5n^2 - mn + 3m - 1 \quad (\text{فصل اول})$$

$$f(0) = ?$$

$$\rightarrow n=0 \rightarrow 2f(0) - 0 = 0 - 0 + 3m - 1 \rightarrow 2f(0) = 3m - 1 \rightarrow f(0) = \frac{3m-1}{2}$$

$$\rightarrow n=-2 \rightarrow 0 + 4f(0) = 14 + 2m + 3m - 1 \rightarrow 4f(0) = 5m + 13$$

$$f(0) = \frac{5m+13}{4}$$

$$\rightarrow \frac{3m-1}{2} = \frac{5m+13}{4} \rightarrow 12m-4 = 5m+13 \rightarrow 7m = 17 \rightarrow m = \frac{17}{7}$$

$$\rightarrow f(0) = \frac{3(\frac{17}{7})-1}{2} = \frac{5(\frac{17}{7})+13}{4} = \frac{17}{7} = 2\frac{3}{7}$$

$$f(n) = an + b \quad (\text{سوال دوم})$$

$$f(n) + f\left(\frac{1}{n}\right) = \frac{3n^2 - 12n + 3}{n} \rightarrow an + b + \frac{a}{n} + b = \frac{3n^2 - 12n + 3}{n}$$

$$\rightarrow an + \frac{a}{n} + 2b = 3n - 12 + \frac{3}{n}$$

$$a=3, b=-9 \rightarrow f(-1) = 3(-1) - 9 \rightarrow f(-1) = -9$$

$$\begin{cases} f(n) = (n-1)\sqrt{1-n} \rightarrow 1-n \geq 0 \rightarrow n \leq 1 \rightarrow Df = (-\infty, 1] \\ g(n) = \sqrt{(1-n)^3} \rightarrow (1-n)^3 \geq 0 \rightarrow 1-n \geq 0 \rightarrow n \leq 1 \rightarrow Dg = (-\infty, 1] \end{cases} \quad (\text{فصل اول})$$

$$Df = Dg \text{ صحیح است } \checkmark (1)$$

$$\text{نکته} \rightarrow g(n) = -\sqrt{(1-n)^2(1-n)} = -|1-n|\sqrt{1-n} \xrightarrow{\text{چون } 1-n \geq 0} g(n) = (n-1)\sqrt{1-n}$$

$$g(n) = (n-1)\sqrt{1-n} \rightarrow f(n) = g(n) \checkmark (2)$$

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$$\begin{cases} f(x) = x\sqrt{x} \rightarrow x \geq 0 \rightarrow Df = [0, +\infty) \\ g(x) = \sqrt{x^3} \rightarrow x^3 \geq 0 \rightarrow x \geq 0 \rightarrow Dg = [0, +\infty) \end{cases} \rightarrow Df = Dg \quad \text{(New)}$$

بالطبع $\rightarrow g(x) = \sqrt{x^2 \cdot x} = |x|\sqrt{x} \xrightarrow{\text{بالطبع}} g(x) = x\sqrt{x} \rightarrow$

$$f(x) = g(x) \quad \text{(2)}$$

$$\begin{cases} f(x) = [\sqrt{x} - [\sqrt{x}]] \rightarrow x \geq 0 \rightarrow [a - [a]] \rightarrow 0 \leq a - [a] < 1 \quad \text{(New)} \\ \rightarrow [a - [a]] = 0 \rightarrow D = \mathbb{R} \rightarrow Df = [0, +\infty) \cap (-\infty, +\infty) \rightarrow Df = [0, +\infty) \\ g(x) = \sqrt{x - |x|} \rightarrow x - |x| \geq 0 \rightarrow x \geq |x| \rightarrow x \geq 0 \rightarrow Dg = [0, +\infty) \end{cases}$$

$$Df = Dg \quad \text{(1)}$$

بالطبع $\rightarrow g(x) = \sqrt{x - x} \rightarrow g(x) = 0$ (بما $x \geq 0$, $f(x) = [\sqrt{x} - [\sqrt{x}]]$)

$$\rightarrow [\sqrt{x}] = \mathbb{Z} \rightarrow f(x) = [\sqrt{x} - \mathbb{Z}]$$

$$\Rightarrow f(x) = [\sqrt{x}] - \mathbb{Z} \rightarrow f(x) = \mathbb{Z} - \mathbb{Z} \rightarrow f(x) = 0 \text{ (بما } x \geq 0)$$

$$\rightarrow f(x) = g(x) \text{ بالتمام } \quad \text{(2)}$$

$$f = \{(1, 2), (2, 3), (3, 1), (0, 0)\}$$

(سوال ۹)

$$g = \{(-1, 1), (1, 1), (2, 3), (3, 0), (0, 2), (\sqrt{2}, 1)\}$$

$$\left(\frac{f+g}{f-g}\right)(1) \rightarrow \frac{f(1) + g(1)}{f(1) - g(1)} = \frac{2+1}{2-1} = 3$$

(الف)

$$\left(\frac{f+g}{f-g}\right)(2) \rightarrow \frac{f(2) + g(2)}{f(2) - g(2)} = \frac{3+3}{3-3} = \frac{6}{0} \rightarrow \text{تعریف نشده}$$

(ب)

$$f(0) \cdot g(-1) = 0 \times 1 = 0$$

(ج)

$$f(m) = \{(2, 1), (-1, 4), (0, 0), (-2, 2), (3, -1)\}$$

(سوال ۱۰)

$$g(n) = f(3n-1) \rightarrow 3n-1=2 \rightarrow 3n=3 \rightarrow n=1$$

(الف)

$$3n-1=-1 \rightarrow 3n=0, n=0, 3n-1=0 \rightarrow 3n=1 \rightarrow n=\frac{1}{3}$$

$$3n-1=-2 \rightarrow 3n=-1 \rightarrow n=-\frac{1}{3}, 3n-1=3 \rightarrow 3n=4, n=\frac{4}{3}$$

$$g(n) = \{(1, 1), (0, 4), (\frac{1}{3}, 0), (-\frac{1}{3}, 2), (\frac{4}{3}, -1)\}$$

$$f(n) = f(|n|) \rightarrow |n|=2 \rightarrow \boxed{n=2 \text{ و } n=-2}, |n|=1, |n|=0$$

(ب)

$$|n|=0 \rightarrow \boxed{n=0} \quad , \quad |n|=-1 \text{ و } |n|=-2 \text{ غلط}$$

$$|n|=3 \rightarrow \boxed{n=3 \text{ و } n=-3}$$

$$f(n) = \{(2, 1), (-2, 1), (0, 0), (3, -1), (-3, -1)\}$$