

$$f(x) = \begin{cases} (a+1)(x+1) & x > 1 \\ 3a+2x & x \leq 1 \end{cases} \quad 2f(a) = f(-1) + a \rightarrow 2(3a+2) = 3a - 1 + a \Rightarrow$$

$$13a+4 = 4a-1 \rightarrow 10a = -5 \rightarrow a = -1/2$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$$

$$f(2-\sqrt{3}) + f(2+\sqrt{3}) \rightarrow \sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2 + \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2 =$$

$$2\sqrt{2-\sqrt{3}} + 2\sqrt{2+\sqrt{3}} + 4 \rightarrow \sqrt{2-\sqrt{3}} = \frac{\sqrt{3}-1}{2}, \sqrt{2+\sqrt{3}} = \frac{\sqrt{3}+1}{2} \Rightarrow$$

$$2\left(\frac{\sqrt{3}-1 + \sqrt{3}+1}{2}\right) + 4 = 2\left(\frac{2\sqrt{3}}{2}\right) + 4 = 2\sqrt{3} + 4$$

$$2f(x) - 3f(-x) = 4x^2 - x \xrightarrow{x^2} 4f(x) - 9f(-x) = 16x^2 - 2x$$

$$2f(-x) - 3f(x) = 4x^2 + x \xrightarrow{x^2} 4f(-x) - 9f(x) = 16x^2 + 3x \rightarrow -5f(x) = 16x^2 + x$$

$$\Rightarrow f(x) = \frac{16x^2 + x}{-5} = -\frac{16}{5}x^2 - \frac{1}{5}x$$

$$(n+1)f(n) - 2nf(n+1) = 4n^2 - mn + 2m - 1$$

$$f(0) = \frac{4 \times 9}{4} - 1 = \frac{36}{4} - 1 = 9 - 1 = 8$$

$$n=0 \rightarrow 2f(0) = 2m - 1 \Rightarrow f(0) = \frac{2m-1}{2}$$

$$n=1 \rightarrow 4f(1) = 16 + 2m + 2m - 1 = 16 + 4m - 1 \Rightarrow f(1) = \frac{15 + 4m}{4}$$

$$\frac{2m-1}{2} = \frac{15 + 4m}{4} \Rightarrow 2m - 1 = 15 + 2m \Rightarrow 16 = 0 \Rightarrow m = \frac{16}{4} = 4$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{4x^2 - 12x + 3}{x} = \frac{4x^2}{x} - \frac{12x}{x} + \frac{3}{x} = 4x - 12 + \frac{3}{x} = 4\left(x + \frac{1}{x}\right) - 12$$

$$2f(-1) = 4(-1) - 12 \rightarrow 2f(-1) = -16 \rightarrow f(-1) = -8$$

$$\begin{cases} f(x) = (x-1)\sqrt{1-x} & D_f: 1-x \geq 0 \rightarrow x \leq 1 \rightarrow [-\infty, 1] \end{cases}$$

$$\begin{cases} g(x) = -\sqrt{(1-x)^3} & D_g: (1-x)^3 \geq 0 \rightarrow 1-x \geq 0 \rightarrow x \leq 1 \rightarrow [-\infty, 1] \end{cases}$$

$$g(x) = -\sqrt{(1-x)^3 \times (1-x)} = -|1-x| \times \sqrt{1-x} \xrightarrow{x \leq 1} -(1-x) \times \sqrt{1-x} =$$

$$(x-1) \times \sqrt{1-x} \stackrel{\text{مطلوب}}{\Rightarrow} f(x) = g(x)$$

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$$\begin{cases} f(x) = x\sqrt{x} \\ g(x) = \sqrt{x^3} \end{cases}$$

$$D_f = x \geq 0 \rightarrow [0, +\infty)$$

$$D_g = x^3 \geq 0 \rightarrow x \geq 0 \rightarrow [0, +\infty)$$

$$D_f = D_g$$

$$g(x) = \sqrt{x^3} = \sqrt{x^2 \cdot x} = |x| \cdot \sqrt{x} \xrightarrow{x \geq 0} g(x) = x\sqrt{x} \Rightarrow g(x) = f(x)$$

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$$\begin{cases} f(x) = [\sqrt{x} - [\sqrt{x}]] \\ g(x) = \sqrt{x - |x|} \end{cases}$$

$$D_f = x \geq 0 \rightarrow [0, +\infty)$$

$$D_g = x - |x| \geq 0 \rightarrow x \geq |x| \xrightarrow{\substack{\sqrt{} \\ -x}} \rightarrow [0, +\infty)$$

$$D_f = D_g$$

$$g(x) \stackrel{x \geq 0}{=} x - |x| = 0 \rightarrow g(x) = 0$$

$$f(x) \stackrel{x \geq 0}{=} 0$$

$$g(x) = f(x)$$

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$$f = \{(1, 2), (2, 3), (3, 1), (0, 0)\} \quad g = \{(-1, 1), (1, 1), (2, 3), (3, 5), (0, 2), (\sqrt{2}, 1)\}$$

$$\text{الف) } \left(\frac{f+g}{f-g} \right)(1) = \frac{f(1)+g(1)}{f(1)-g(1)} = \frac{2+1}{2-1} = \frac{3}{1} = 3$$

$$\text{ب) } \left(\frac{f+g}{f-g} \right)(2) = \frac{f(2)+g(2)}{f(2)-g(2)} = \frac{3+3}{3-3} = \frac{6}{0} \Rightarrow \text{تعیّن}$$

$$\text{ج) } f(0)g(-1) = 0 \times 1 = 0$$

$$f(x) = \{(2, 1), (-1, 4), (0, 5), (-2, 2), (3, -1)\} \quad g(x) = \{(1, 1), (0, 4), (\frac{1}{4}, 5), (-\frac{1}{4}, 2), (\frac{4}{3}, -1)\}$$

$$\text{الف) } g(x) = f(3x-1)$$

$$\begin{cases} 3x-1=2 \rightarrow 3x=3 \rightarrow x=1 \\ 3x-1=-1 \rightarrow 3x=0 \rightarrow x=0 \\ 3x-1=0 \rightarrow 3x=1 \rightarrow x=\frac{1}{3} \\ 3x-1=-2 \rightarrow 3x=-1 \rightarrow x=-\frac{1}{3} \\ 3x-1=3 \rightarrow 3x=4 \rightarrow x=\frac{4}{3} \end{cases}$$

$$\begin{cases} g(1) = f(2) = 1 \\ g(0) = f(-1) = 4 \\ g(\frac{1}{3}) = f(0) = 5 \\ g(-\frac{1}{3}) = f(-2) = 2 \\ g(\frac{4}{3}) = f(3) = -1 \end{cases}$$

$$\text{ب) } h(x) = f(|x|)$$

$$\begin{cases} h(2) = f(2) = 1 \\ h(-1) = f(1) = 4 \\ h(0) = f(0) = 5 \\ h(-2) = f(2) = 1 \\ h(3) = f(3) = -1 \end{cases}$$

$$h(x) = \{(2, 1), (0, 5), (-2, 1), (3, -1)\}$$