

$$f(x) = \begin{cases} (a+1)(x+1) & ; x > 1 \xrightarrow{f(1)} (a+1)(1+1) \Rightarrow \underline{2a+2} \\ 2a+2x & ; x \leq 1 \xrightarrow{f(-1)} 2a-2 \end{cases}$$

$$f(f(1)) = f(2a+2) = 2a-2 \Rightarrow 2(2a+2) = 2a-2 \Rightarrow 4a+4 = 2a-2 \Rightarrow 2a = -6 \Rightarrow a = -3$$

$$a = -3$$

$$\boxed{a = -3, 1}$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \Rightarrow \begin{cases} ① \sqrt{x-\sqrt{x}} + \frac{1}{\sqrt{x-\sqrt{x}}} + 2 \\ ② \sqrt{x+\sqrt{x}} + \frac{1}{\sqrt{x+\sqrt{x}}} + 2 \end{cases} \Rightarrow \sqrt{x-\sqrt{x}} + \frac{1}{\sqrt{x-\sqrt{x}}} + \sqrt{x+\sqrt{x}} + \frac{1}{\sqrt{x+\sqrt{x}}} + 4$$

$$f(x-\sqrt{x}) + f(x+\sqrt{x}) = ? \quad \boxed{\sqrt{4}+4}$$

$$(x-\sqrt{x})(x+\sqrt{x}) = 1$$

$$\sqrt{x-\sqrt{x}} = \frac{1}{\sqrt{x+\sqrt{x}}} \quad / \quad \sqrt{x+\sqrt{x}} = \frac{1}{\sqrt{x-\sqrt{x}}}$$

$$\begin{aligned} & (\sqrt{x-\sqrt{x}} + \sqrt{x+\sqrt{x}})^2 \\ &= x-\sqrt{x} + x+\sqrt{x} + 2\sqrt{(x-\sqrt{x})(x+\sqrt{x})} \\ &= 2x + 2\sqrt{1} = 2x + 2 \end{aligned}$$

$$\boxed{\sqrt{4}+4}$$

$$f(x) - f(-x) = 2x^2 - x$$

$$\underline{x = -x} \quad f(-x) - f(x) = 2x^2 + x \quad \left\{ \begin{array}{l} ① -f(-x) - f(x) = 12x^2 \Rightarrow f(-x) + f(x) = 12x^2 \\ ② -f(x) + f(-x) = -12x^2 \end{array} \right.$$

$$\underline{x = x} \quad f(x) - f(-x) = 2x^2 - x$$

$$③ -f(x) + f(-x) = -12x^2$$

$$-a f(-x) + a f(x) = -12x^2$$

$$\underline{f(-x) + f(x) = \frac{12x^2}{a}}$$

$$a+b = 12f(x) = -12x^2 - \frac{12x}{a} \Rightarrow \underline{f(x) = -x^2 - \frac{x}{a}}$$

$$(x+1)f(x) - xf(x+1) = 2x^2 - mx + m - 1$$

$$\underline{f(0)} \Rightarrow f(0) = m-1$$

$$f(0) = \frac{m-1}{1}$$

$$\underline{f(1)} \Rightarrow 4f(0) = 12 + 2m$$

$$\frac{4(m-1)}{1} = 12 + 2m \Rightarrow 4m - 4 = 12 + 2m \Rightarrow 2m = 16 \Rightarrow \underline{m = 8}$$

$$f(0) = \frac{8 \times \frac{9}{8} - 1}{1} = \frac{9 - 1}{1} = \frac{8}{1} = \underline{\frac{8}{1}}$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{2x^2 - 12x + 1}{x} \Rightarrow 2x - 12 + \frac{1}{x} \xrightarrow{ax+b} a\left(x+\frac{1}{x}\right) + b$$

$$\boxed{a = 2}$$

$$\boxed{b = -12}$$

$$f(x) = ax + b \Rightarrow f(-1) = -2 - 12 = \underline{-14}$$

$$\begin{cases} f(x) = (x-1)\sqrt{1-x} \Rightarrow x-1 = -(-x) \Rightarrow f(x) = -(1-x)^{\frac{3}{2}} \\ g(x) = -\sqrt{(1-x)^3} \Rightarrow -(1-x)^{\frac{3}{2}} \end{cases}$$

$$g(x) = f(x)$$

$$\begin{cases} f(x) = x\sqrt{x} \Rightarrow \sqrt{x^2}\sqrt{x} \rightarrow \text{چون داریم } x \geq 0 \rightarrow \sqrt{x^2} = x \\ g(x) = \sqrt{x^3} \Rightarrow \sqrt{x^2 \cdot x} = x\sqrt{x} \end{cases}$$

$$g(x) = f(x)$$

$$\begin{cases} f(x) = [\sqrt{x} - \lfloor \sqrt{x} \rfloor] \Rightarrow \text{چون داخل کروشه بين صفر و يك است: } f(x) = 0 \\ g(x) = \sqrt{x - \lfloor x \rfloor} \Rightarrow \text{چون } x \geq 0 \Rightarrow x - \lfloor x \rfloor = 0 \Rightarrow g(x) = 0 \end{cases}$$

داينه f نقطه x را نشانه f
چون x > 0 است
↓
f(x) = g(x)

$$f = \{(1, 2), (2, 3), (3, 1), (0, 0)\}$$

$$\text{اين: } \left[\frac{f+g}{f-g} \right] (1) = \frac{2+1}{2-1} = 3$$

$$g = \{(-1, 1), (1, 1), (2, 3), (3, 2), (0, 2), (1, 2)\}$$

$$\text{ب: } \left[\frac{f+g}{f-g} \right] (2) = \frac{3+2}{3-2} = \frac{5}{1} \quad \text{تكرار نموده}$$

$$\text{ج: } f(0)g(1) = 0 \times 1 = 0$$

$$f(x) = \{(2, 1), (-1, 2), (0, 2), (2, 2), (3, -1)\}$$

$$\text{اين: } g(x) = f(x-1) \Rightarrow g(x) = \{(1, 1), (0, 2), (1, 2), (2, 2), (2, -1)\}$$

$$\text{ب: } h(x) = f(1-x) \Rightarrow h(x) = \{(1, 1), (2, 2), (0, 2), (2, -1), (2, -1)\}$$