

$$f(n) = \begin{cases} (n+1)(n+2) & n \geq 1 \\ 3n+2 & n \leq 1 \end{cases} \quad \textcircled{*} \quad 2f(0) = f(-1) + a \Rightarrow 2 \times (n+1)(n) = 3a - 1 + a$$

$$1 \times a + 1 \times 2 = 3a - 1 \Rightarrow 10a = -1 \quad \boxed{a = -\frac{1}{10}}$$

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$$f(n) = \sqrt{n} + \frac{1}{\sqrt{n}} + 2$$

$$f(\sqrt{2}) + f(\sqrt{2}) = \sqrt{2} + \frac{1}{\sqrt{2}} + 2 + \sqrt{2} + \frac{1}{\sqrt{2}} + 2 = 2\sqrt{2} + \frac{2}{\sqrt{2}} + 4 = 2\sqrt{2} + \sqrt{2} + 4 = 3\sqrt{2} + 4$$

$$\frac{f(\sqrt{2}) + f(\sqrt{2})}{2} = \frac{3\sqrt{2} + 4}{2} = \frac{3\sqrt{2}}{2} + 2$$

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$$2f(n) - 3f(-n) = \varepsilon n^2 - n$$

$$\begin{aligned} \text{for } n &\rightarrow 2f(n) - 3f(-n) = \varepsilon n^2 - n \xrightarrow{\times 2} 4f(n) - 6f(-n) = 2\varepsilon n^2 - 2n \\ \text{for } -n &\rightarrow 2f(-n) - 3f(n) = \varepsilon n^2 + n \xrightarrow{\times 2} 4f(-n) - 6f(n) = 2\varepsilon n^2 + 2n \end{aligned}$$

$$\begin{aligned} &4f(n) - 6f(-n) = 2\varepsilon n^2 - 2n \\ &4f(-n) - 6f(n) = 2\varepsilon n^2 + 2n \\ \hline &-2f(n) = 2\varepsilon n^2 + 4n \\ &\Rightarrow \boxed{f(n) = -\varepsilon n^2 - \frac{1}{2}n} \end{aligned}$$

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$$(n+1)f(n) - nm f(n+1) = \varepsilon n^2 - mn + m-1$$

$$n=0 \quad 1f(0) = 0m-1 \Rightarrow 1f(0) = m-1$$

$$n=-1 \quad 4f(0) = 14 + 2m + m-1 \Rightarrow 4f(0) = 14 + 3m-1 \Rightarrow 14 + 3m-1 = 9m-1$$

$$\Rightarrow 1f(0) = \frac{14}{4} - 1 = \frac{10}{4} \Rightarrow \boxed{f(0) = \frac{5}{2} = \frac{10}{4}} \quad m = 18 \Rightarrow m = \frac{18}{\varepsilon}$$

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$$f(n) + f\left(\frac{1}{n}\right) = 3n^2 - 12n + 4$$

$$f(n) = an + b \quad \text{از } f(n) = an + b$$

$$\Rightarrow an + b + \frac{a}{n} + b = \frac{3n^2 - 12n + 4}{n} \Rightarrow an^2 + \frac{a}{n} + 2b = \frac{3n^2 - 12n + 4}{n}$$

$$\Rightarrow an^2 + a + 2bn = 3n^2 - 12n + 4$$

$$\Rightarrow a = 3 \quad 2b = -12 \Rightarrow b = -6$$

$$\Rightarrow f(n) = 3n - 6$$

$$f(-1) = -3 - 6 = \boxed{-9}$$

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$$\begin{cases} f(n) = (n-1)\sqrt{1-n} \\ g(n) = -\sqrt{(1-n)^3} \end{cases} \rightarrow \begin{cases} n \leq 1 \Rightarrow Df = (-\infty, 1] \\ n \leq 1 \Rightarrow Dg = (-\infty, 1] \end{cases} \Rightarrow Df = Dg$$

$$\rightarrow n = 0 \Rightarrow \begin{cases} f(n) = -1 \\ g(n) = -1 \end{cases} \quad n = -1 \Rightarrow \begin{cases} f(n) = -1 \\ g(n) = -1 \end{cases} \Rightarrow f(n) = g(n)$$

مساوی ✓
 در این حالت مساوی است و در هر دو طرف مساوی است

$$\begin{cases} f(n) = n\sqrt{n} \\ g(n) = \sqrt{n^3} \end{cases} \rightarrow \begin{cases} n \geq 0 \Rightarrow Df = [0, +\infty) \\ n \geq 0 \Rightarrow Dg = [0, +\infty) \end{cases} \Rightarrow Df = Dg$$

$$\rightarrow n = 1 \Rightarrow \begin{cases} f(n) = 1 \cdot \sqrt{1} = 1 \\ g(n) = \sqrt{1^3} = 1 \end{cases} \quad n = \frac{1}{2} \Rightarrow \begin{cases} f(n) = \frac{1}{2}\sqrt{\frac{1}{2}} \\ g(n) = \sqrt{\frac{1}{2^3}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4} \end{cases}$$

$$\Rightarrow f(n) = g(n)$$

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$$\begin{cases} f(n) = \lfloor \sqrt{n} \rfloor - \lfloor \sqrt{n} \rfloor \\ g(n) = \sqrt{n - \ln n} \end{cases} \rightarrow \begin{cases} n \geq 0 \Rightarrow Df = [0, +\infty) \\ n \geq 0 \Rightarrow Dg = [0, +\infty) \end{cases} \Rightarrow Df = Dg$$

$$Rg = 0 \rightarrow \text{در این حالت مساوی است}$$

$$f(n) = \lfloor \sqrt{n} \rfloor - \lfloor \sqrt{n} \rfloor = 0 \Rightarrow f(n) = g(n)$$

در این حالت مساوی است و در هر دو طرف مساوی است ✓

$$f = \{(1, 2), (2, 2), (2, 1), (0, 0)\} \quad g = \{(-1, 1), (1, 1), (2, 2), (2, 2), (0, 2), (2, 1), (2, 1)\}$$

الف) $\left(\frac{f+g}{f-g}\right)(1) = \frac{f(1)+g(1)}{f(1)-g(1)} = \frac{2+1}{2-1} = 3$

ب) $\left(\frac{f+g}{f-g}\right)(2) = \frac{f(2)+g(2)}{f(2)-g(2)} = \frac{2+2}{2-2} \rightarrow \text{مخرج 0} \Rightarrow \text{تعریف نشده}$

ج) $f(0)g(-1) = 0 \times 1 = 0$

$$f(n) = \{(2, 1), (-1, 2), (0, 0), (-2, 2), (3, -1)\}$$

$$g(n) = f(3n-1) \Rightarrow g(n) = \{(1, 1), (0, 2), (1, 2), (2, 2), (8, -1)\}$$

د) $h(n) = f(3n-1) \Rightarrow h(n) = \{(2, 1), (-1, 2), (0, 0), (3, -1), (-3, -1)\}$

$$|x| = 2 \rightarrow x = 2 \text{ یا } x = -2, |x| = -1 \rightarrow \text{غیرممکن}, |x| = -2 \rightarrow \text{غیرممکن}$$