

$$\textcircled{1} f(x) = f(-x) + a \quad f(x) = \begin{cases} (a+1)(x+2) & x > 1 \\ 3a + 2x & x \leq 1 \end{cases}$$

$$f(x) = (a+1)(x+2) = va + v$$

$$f(-x) = 3a + 2(-x) = 3a - 2$$

$$xf(x) = f(-x) + a \Rightarrow x(va + v) = 3a - 2 + a \Rightarrow \textcircled{2}$$

$$11a + 12 = 4a - 2 \Rightarrow 7a = -14 \Rightarrow a = -2$$

$$\textcircled{2} f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \quad f(x-\sqrt{x}) + f(x+\sqrt{x}) = ?$$

$$(x-\sqrt{x})(x+\sqrt{x}) = x - x = 0 \rightarrow \text{این دو عدد یکدیگر را خنثی می‌کنند}$$

$$f(x_1) = \sqrt{x_1} + \frac{1}{\sqrt{x_1}} + 2 \Rightarrow \sqrt{x_1} + \sqrt{x_2} + 2 = f(x_1)$$

$$f(x_2) = \sqrt{x_2} + \frac{1}{\sqrt{x_2}} + 2 \Rightarrow f(x_2) = \sqrt{x_2} + \sqrt{x_1} + 2 \quad \textcircled{2}$$

$$f(x_1) + f(x_2) = 2(\sqrt{x_1} + \sqrt{x_2}) + 4 = 2(\sqrt{4}) + 4 = 4 + 2\sqrt{4}$$

$$(\sqrt{x_1} + \sqrt{x_2}) = ? \Rightarrow (\sqrt{x_1} + \sqrt{x_2})^2 = (\sqrt{x-\sqrt{x}} + \sqrt{x+\sqrt{x}})^2$$

$$= (x-\sqrt{x})(x+\sqrt{x}) + 2\sqrt{(x-\sqrt{x})(x+\sqrt{x})} = 0 + 2\sqrt{x} = 2\sqrt{x}$$

$$= 4 + 4 = 8$$

$$(\sqrt{x_1} + \sqrt{x_2}) = \sqrt{8}$$

$$③ \quad rf(x) - rf(-x) = rx^2 - x$$

$$\left\{ \begin{array}{l} rf(x) - rf(-x) = rx^2 - x \\ rf(-x) - rf(x) = rx^2 + x \end{array} \right.$$

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$$- \omega f(x) = rx^2 + x \Rightarrow f(x) = \frac{rx^2 - x}{\omega}$$

$$\Rightarrow f(x) = -rx^2 - \frac{1}{\omega}x$$

$$④ \quad (x+r)f(x) - rf(x+r) = rx^2 - mx + rm - 1$$

$$\xrightarrow{x=0} rf(0) = rm - 1 \Rightarrow \frac{rm - 1}{r} = \frac{1\omega + \omega m}{\cancel{r}}$$

$$\xrightarrow{x=-r} rf(0) = 1\omega + \omega m$$

$$4m - r = 1\omega + \omega m \Rightarrow rm = 1 \Rightarrow m = \frac{1}{r} = r\omega$$

$$f(0) = \frac{r(r\omega) - 1}{r} = \frac{r^2\omega - 1}{r} = \frac{r\omega}{r} = \frac{r\omega}{r}$$

$$\textcircled{a} \quad P(-1) = ? \quad P(x) + P\left(\frac{1}{x}\right) = \frac{x^2 - 1x + 1}{x}$$

$$\begin{aligned} \xrightarrow{x=-1} P(-1) + P(-1) &= \frac{1 + 1 + 1}{-1} \Rightarrow 2P(-1) = -1 \\ \Rightarrow P(-1) &= -\frac{1}{2} \end{aligned}$$

$$\textcircled{b} \quad P(x) = (x-1)\sqrt{1-x}$$

$$g(x) = -\sqrt{(1-x)^3}$$

$$1-x \geq 0 \Rightarrow 1 \geq x$$

$$x-1 \in \mathbb{R} \Rightarrow x \in \mathbb{R} \quad \left. \begin{array}{l} \text{ } \\ \text{ } \end{array} \right\} \cap \Rightarrow 1 \geq x$$

$$P_f = P_g$$

$$(1-x)^3 \geq 0 \Rightarrow 1-x \geq 0 \Rightarrow 1 \geq x$$

$$x=1 \Rightarrow P(x)=0, g(x)=0$$

Original (Jamil 10/2/20)

$$x=0 \Rightarrow P(x)=1, g(x)=-1$$

$$P(x) = g(x)$$

$$\textcircled{v} \quad f(x) = x\sqrt{x}$$

$$g(x) = \sqrt{x^3}$$

$$x \in \mathbb{R}$$

$$x \geq 0$$

$$x \geq 0$$

$$x^3 \geq 0 \Rightarrow x \geq 0$$

$$D_f = D_g$$

$$g(x) = f(x)$$

برای هر $x \geq 0$ داریم
 $f(x) = g(x)$

$$\textcircled{vi} \quad f(x) = [\sqrt{x} - [\sqrt{x}]]$$

$$g(x) = \sqrt{x - |x|}$$

$$g(x) = f(x)$$

$$x - |x| \geq 0 \Rightarrow x \geq |x| \Rightarrow x \geq 0$$

$$x \geq 0 \Rightarrow D_f = D_g = [0, +\infty)$$

برای هر $x \geq 0$ داریم

$$\textcircled{a} \quad \text{الف) } \left(\frac{f+g}{f-g} \right)(1) = \frac{f(1)+g(1)}{f(1)-g(1)} = \frac{1+1}{1-1} = \frac{2}{0} = \text{مفرد}$$

$$\text{ب) } \left(\frac{f+g}{f-g} \right)(2) = \frac{f(2)+g(2)}{f(2)-g(2)} = \frac{2+2}{2-2} = \frac{4}{0} = \text{مفرد}$$

$$\text{ج) } f(0)g(-1) = 0 \cdot 1 = 0$$

⑩

$$f) g(x) = F(3x-1)$$

$$1 = 3x-1 \Rightarrow x=0 \Rightarrow g(0) = F(1) = 1 \Rightarrow (0, 1)$$

$$-1 = 3x-1 \Rightarrow x=0 \Rightarrow g(0) = F(-1) = 2 \Rightarrow (0, 2)$$

$$0 = 3x-1 \Rightarrow x = \frac{1}{3} \Rightarrow g(\frac{1}{3}) = F(0) = 0 \Rightarrow (\frac{1}{3}, 0)$$

$$-2 = 3x-1 \Rightarrow x = -\frac{1}{3} \Rightarrow g(-\frac{1}{3}) = F(-2) = 2 \Rightarrow (-\frac{1}{3}, 2)$$

$$3 = 3x-1 \Rightarrow x = \frac{4}{3} \Rightarrow g(\frac{4}{3}) = F(3) = -1 \Rightarrow (\frac{4}{3}, -1)$$

$$\{(0, 1), (0, 2), (\frac{1}{3}, 0), (-\frac{1}{3}, 2), (\frac{4}{3}, -1)\}$$

$$ج) h(x) = F(|x|)$$

$$|x|=0 \Rightarrow x=0 \Rightarrow h(0) = F(0) = 0 \Rightarrow (0, 0)$$

$$|x|=1 \Rightarrow x=1 \Rightarrow h(1) = F(1) = 1 \Rightarrow (1, 1)$$

$$x=-1 \Rightarrow h(-1) = F(1) = 1 \Rightarrow (-1, 1)$$

$$|x|=2 \Rightarrow x=2 \Rightarrow h(2) = F(2) = -1 \Rightarrow (2, -1)$$

$$x=-2 \Rightarrow h(-2) = F(2) = -1 \Rightarrow (-2, -1)$$

$$\{(0, 0), (1, 1), (-1, 1), (2, -1), (-2, -1)\}$$