

$$f(n) = \begin{cases} (a+1)(n+2) & , n > 1 \\ 2a+2n & , n \leq 1 \end{cases}$$

(1)

$$2f(w) = f(-2) + a \quad a = ?$$

$$\left. \begin{aligned} f(w) &= (a+1) \rightarrow 1 \cdot a + 1 \\ f(-2) &= 2a - 2 \end{aligned} \right\} \begin{aligned} 1 \cdot a + 1 &= 2a + a - 2 \Rightarrow 1 \cdot a - a = -1 \Rightarrow a = -1 \end{aligned} \rightarrow \boxed{a = -1}$$

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$$f(n) = \sqrt{2n} + \frac{1}{\sqrt{2n}} + 2$$

(2)

$$(2+\sqrt{2})(2-\sqrt{2}) = 4-2=1 \rightarrow \frac{1}{\sqrt{2+\sqrt{2}}} = \sqrt{2-\sqrt{2}}$$

$$f(2+\sqrt{2}) = \sqrt{2+\sqrt{2}} + \sqrt{2-\sqrt{2}} + 2 = \sqrt{2+\sqrt{2}} + \sqrt{2-\sqrt{2}} + 2 = 4 + 2\sqrt{1} = 4 + 2 = 6 \rightarrow \sqrt{6}$$

$$f(2-\sqrt{2}) \text{ و } f(2+\sqrt{2}) = \boxed{6}$$

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$$2f(-n) - 3f(n) = 4n^2 + n$$

$$2f(n) - 3f(-n) = 4n^2 - n \xrightarrow{n=-n} f(-n) - 3f(n) = 4(-n)^2 - (-n) = 4n^2 + n$$

(3)

$$\begin{aligned} \times 2 \rightarrow 2f(-n) - 9f(n) &= 4n^2 + n \\ f(n) - 3f(-n) &= 4n^2 - n \end{aligned} \left\{ \begin{aligned} f(n) - 3f(-n) + 3f(-n) - 9f(n) &= \\ 4n^2 - n + 2n + 2n &= \end{aligned} \right.$$

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$$\boxed{f(n) = -2n^2 - \frac{n}{2}}$$

$$\leftarrow -2f(n) = 4n^2 + n$$

$$(n+2)f(n) - 2nf(n+2) = 4n^2 - mn + 2m - 1$$

(4)

$$f(0) \rightarrow 2f(0) - 4f(2) = 2m - 1$$

$$f(-2) \rightarrow 0 \text{ و } 4f(0) = (4+2m)m-1 \rightarrow f(0) = \frac{2m+1}{4}$$

$$\left. \begin{aligned} \frac{2m+1}{4} - 4f(2) &= 2m-1 \\ f(2) &= \frac{2m-1}{-12} \end{aligned} \right\}$$

$$\frac{2m+1}{4} - \frac{2m-1}{3} = 2m-1 \rightarrow m=0$$

$$\boxed{f(0) = \frac{1}{4}}$$

$$f(-1) = \frac{f(-1) + f(-1)}{2f(-1)} = \frac{1+1+1}{-1} = \boxed{f(-1) = -3} \quad (5)$$

$$\begin{aligned} f(n) &= (n-1)\sqrt{1-n} \quad n \leq 1 \\ g(n) &= -\sqrt{(1-n)^3} \quad 1-n \geq 0 \rightarrow n \leq 1 \end{aligned} \quad \left. \begin{aligned} D_f &= n \leq 1 \\ D_g &= n \leq 1 \end{aligned} \right\} \quad \begin{aligned} -\sqrt{(1-n)^3} &= -\sqrt{(1-n)} \times \sqrt{(1-n)^2} \\ &= -|1-n| \times \sqrt{(1-n)} = (n-1)\sqrt{1-n} \end{aligned} \quad (6)$$

$$\boxed{f(n) = g(n)} \quad (6)$$

$$\begin{aligned} f(n) &= n\sqrt{n} \rightarrow D_f = n \geq 0 \\ g(n) &= \sqrt{n^3} \rightarrow D_g = n \geq 0 \rightarrow n \geq 0 \end{aligned} \quad \begin{aligned} \sqrt{n^3} &= |n|\sqrt{n} \\ &= n\sqrt{n} \end{aligned} \quad (7)$$

$$\boxed{f(n) = g(n)} \quad (7)$$

$$\begin{aligned} f(n) &= [\sqrt{n} - \lfloor \sqrt{n} \rfloor] \rightarrow n \geq 0 \\ g(n) &= \sqrt{n - \lfloor n \rfloor} \rightarrow n - \lfloor n \rfloor \geq 0 \end{aligned} \quad \begin{aligned} D_f &\neq D_g \\ &\text{مثلاً } n=2 \text{ جواب نشود!} \end{aligned} \quad (8)$$

$$\boxed{f(n) \neq g(n)} \quad (8)$$

$$\lim_{x \rightarrow 1} \left(\frac{f+g}{f-g} \right)(1) = \frac{f(1)+g(1)}{f(1)-g(1)} = \frac{2+1}{2-1} = \boxed{3} \quad (9)$$

$$\lim_{x \rightarrow 2} \left(\frac{f+g}{f-g} \right)(2) = \frac{f(2)+g(2)}{f(2)-g(2)} = \frac{2+2}{2-2} = \frac{4}{0} \rightarrow \boxed{\text{تقریب نشده!}}$$

$$2.) = f(0)g(-1) \rightarrow 0 \times 1 = \boxed{0}$$

$$\lim_{n \rightarrow \infty} g(n) = f(2n-1) \rightarrow \begin{aligned} 2 &\rightarrow 2n-1=2 \rightarrow n=1.5 \rightarrow \text{نه} \\ -1 &\rightarrow 2n-1=-1 \rightarrow n=0 \rightarrow \text{نه} \\ 0 &\rightarrow 2n-1=0 \rightarrow n=0.5 \rightarrow \text{نه} \\ -2 &\rightarrow 2n-1=-2 \rightarrow n=-0.5 \rightarrow \text{نه} \\ 3 &\rightarrow 2n-1=3 \rightarrow n=2 \rightarrow \text{نه} \end{aligned} \quad (10)$$

$$\boxed{g(n) = \{f(2n-1)\}} \quad (10)$$

$$\begin{aligned} \lim_{n \rightarrow \infty} h(n) &= f(|n|) \\ |n| &= 2 \rightarrow n = \pm 2 \rightarrow 2, -2 \\ |n| &= 0 \rightarrow n = 0 \rightarrow 0 \end{aligned} \quad \begin{aligned} |n| &= 3 \rightarrow n = \pm 3 \rightarrow 3, -3 \\ |n| &= 1 \rightarrow n = \pm 1 \rightarrow 1, -1 \end{aligned}$$

$$\boxed{h(n) = \{(2,1), (-2,1), (0,0), (2,-1), (-2,-1)\}} \quad (11)$$

$$2f(n) - 3f(-n) = 4n^2 - n \xrightarrow{x^2} 4f(n) - 9f(-n) = 16n^2 - 2n \quad (3)$$

$$2f(-n) - 3f(n) = 4(-n)^2 + n \xrightarrow{x^3} 4f(-n) - 9f(n) = (4n^2 + 3n$$

$$- 5f(n) = 4 \cdot n^2 + n \rightarrow \boxed{f(n) = -4n^2 - \frac{n}{5}}$$

$$f(n) = [\sqrt{n}] - [\sqrt{n}] \rightarrow \sqrt{n} \rightarrow n \geq 0 \rightarrow Df = [0, +\infty)$$

$$g(n) = \sqrt{n - |n|} \rightarrow n - |n| \geq 0 \rightarrow Dg = [0, +\infty) \quad (1)$$

$$f(n) = [\sqrt{n}] - [\sqrt{n}] = 0 \quad \text{عدد صحیح از برکت می آید بیرون ت}$$

$$g(n) = 0 \quad \text{آنها با یکدیگر برابرند!} \rightarrow n - |n| = n - n = 0 \quad \text{نسبت صفر است!}$$

۱- الف) یک اگرمی ازیم و طبق آن می‌توانیم رابطه را بنویسیم. f آن‌ها را می‌تواند.

$$w_{n-1} = t \rightarrow n = \frac{t+1}{3} \quad f(w_{n-1}) = \left\{ \left(\frac{t+1}{3}, 1 \right), \left(\frac{-t+1}{3}, 4 \right), \left(\frac{t+1}{3}, 5 \right), \left(\frac{-t+1}{3}, 2 \right), \left(\frac{t+1}{3}, -1 \right) \right\}$$

$$f(w_{n-1}) = \left\{ (1, 1), (0, 4), \left(\frac{1}{3}, 5 \right), \left(-\frac{1}{3}, 2 \right), \left(\frac{4}{3}, -1 \right) \right\}$$