

$$f(n) = \begin{cases} (a+1)(n+1) & , n > 1 \\ \psi a + \psi n & , n \leq 1 \end{cases}$$

(1)

$$\psi f(n) = f(-n) + a \quad a = ?$$

$$\left. \begin{aligned} f(n) &= (a+1)\psi \rightarrow \psi a + \psi n \\ f(-n) &= \psi a - n \end{aligned} \right\} \psi a + \psi n = \psi a + a - n \Rightarrow \psi a = -1 \rightarrow \boxed{a = -1/\psi}$$

(2)

$$\begin{aligned} f(n) &= \sqrt{2n} + \frac{1}{\sqrt{2n}} + \psi \\ (\psi + \sqrt{2})(\psi - \sqrt{2}) &= \psi^2 - 2 = 1 \rightarrow \frac{1}{\sqrt{\psi^2 - 2}} = \sqrt{\psi - \sqrt{2}} \\ f(\psi + \sqrt{2}) &= \sqrt{\psi + \sqrt{2}} + \sqrt{\psi - \sqrt{2}} + \psi \\ &= \psi + \sqrt{2} + \psi - \sqrt{2} + \psi = 3\psi = \sqrt{6} \\ \hookrightarrow \sqrt{6+2} &= f(\psi - \sqrt{2}) \\ f(\psi - \sqrt{2}) &= f(\psi + \sqrt{2}) = \boxed{\sqrt{6+2}} \end{aligned}$$

$$\begin{aligned} f(n) - \psi f(-n) &= \psi n^2 - n \xrightarrow{n=-n} f(-n) - \psi f(n) = \psi(-n)^2 - (-n) \\ &= \psi n^2 + n \\ \xrightarrow{\times \psi} \left. \begin{aligned} \psi f(-n) - \psi^2 f(n) &= \psi^2 n^2 + \psi n \\ f(n) - \psi f(-n) &= \psi n^2 - n \end{aligned} \right\} \begin{aligned} f(n) - \psi f(-n) + \psi f(-n) - \psi^2 f(n) &= \\ \psi n^2 - n + \psi n^2 + \psi n &= \end{aligned} \\ \boxed{f(n) = -\psi n^2 - \frac{n}{\psi}} & \leftarrow -1 f(n) = \psi n^2 + n \end{aligned}$$

$$(n+1)f(n) - \psi n f(n+1) = \psi n^2 - mn + \psi m - 1$$

(4)

$$\begin{aligned} f(0) &\rightarrow \psi f(0) - \psi f(1) = \psi m - 1 \\ f(-1) &\rightarrow 0 + \psi f(0) = (1 + \psi m) - 1 \rightarrow f(0) = \frac{\psi m + 1}{\psi} \end{aligned} \left\} \begin{aligned} \frac{\psi m + 1}{\psi} - \psi f(1) &= \psi m - 1 \\ f(1) &= \frac{\psi m - 1}{-1\psi} \end{aligned} \right.$$

$$\frac{\psi m + 1}{\psi} - \frac{\psi m - 1}{\psi} = \psi m - 1 \rightarrow m = 0$$

$$\boxed{f(0) = \frac{1}{\psi}}$$

$$f(-1) = \frac{f(-1) + f(-1)}{2f(-1)} = \frac{1+1+1}{-1} = \boxed{f(-1) = -3} \quad (5)$$

$$\left. \begin{aligned} f(n) &= (n-1)\sqrt{1-n} \\ g(n) &= -\sqrt{(1-n)^3} \end{aligned} \right\} \begin{aligned} D_f &= n \leq 1 \\ D_g &= n \leq 1 \end{aligned}$$

$1-n \geq 0 \rightarrow n \leq 1$

$-\sqrt{(1-n)^3} = -\sqrt{(1-n)} \times \sqrt{(1-n)^2}$
 $= -|1-n| \times \sqrt{(1-n)} = (n-1)\sqrt{1-n}$

$\boxed{f(n) = g(n)}$

$$\left. \begin{aligned} f(n) &= n\sqrt{n} \\ g(n) &= \sqrt{n^3} \end{aligned} \right\} \begin{aligned} D_f &= n \geq 0 \\ D_g &= n \geq 0 \rightarrow n \geq 0 \end{aligned}$$

$\sqrt{n^3} = |n|\sqrt{n}$
 $n\sqrt{n}$

$\boxed{f(n) = g(n)}$

$$\left. \begin{aligned} f(n) &= [\sqrt{n} - [\sqrt{n}]] \\ g(n) &= \sqrt{n - [n]} \end{aligned} \right\} \begin{aligned} D_f &= n \geq 0 \\ D_g &= n - [n] \geq 0 \rightarrow \mathbb{R} \end{aligned}$$

$D_f \neq D_g$

$\boxed{f(n) \neq g(n)}$

$$1.) \left(\frac{f+g}{f-g} \right)(1) = \frac{f(1)+g(1)}{f(1)-g(1)} = \frac{1+1}{1-1} = \boxed{\infty}$$

$$2.) \left(\frac{f+g}{f-g} \right)(2) = \frac{f(2)+g(2)}{f(2)-g(2)} = \frac{2+2}{2-2} = \frac{4}{0} \rightarrow \boxed{\text{تعريف نشده}}$$

$$2.) = f(0)g(-1) \rightarrow 0 \times 1 = \boxed{0}$$

$$1.) g(n) = f(2n-1) \rightarrow$$

$1 \rightarrow 2n-1=1 \rightarrow n=1 \rightarrow \checkmark$
$-1 \rightarrow 2n-1=-1 \rightarrow n=0 \rightarrow \checkmark$
$0 \rightarrow 2n-1=0 \rightarrow n=\frac{1}{2} \rightarrow \times$
$-2 \rightarrow 2n-1=-2 \rightarrow n=-\frac{1}{2} \rightarrow \times$
$3 \rightarrow 2n-1=3 \rightarrow n=2 \rightarrow \checkmark$

$\boxed{g(n) = \{(0,0)\}}$

$|n| \rightarrow$ در هر بار مثبت

$$\therefore h(n) = f(|n|)$$

$$|n| = 1 \rightarrow n = \pm 1 \rightarrow 1, -1 \quad |n| = 2 \rightarrow n = \pm 2 \rightarrow -1$$

$$|n| = 0 \rightarrow n = 0 \rightarrow 0$$

$$\boxed{h(n) = \{(1,1), (-1,-1), (0,0), (2,1), (-2,-1)\}}$$