

تکلیف ۹

آتریساقا صال

$$2f(a) = f(-2) + a \Rightarrow 2(a+1)(1) = (3a-4) + a \Rightarrow$$

$$\Rightarrow 12a + 12 = 4a - 4 \rightarrow 10a = -16 \quad a = \frac{-16}{10} = -\frac{8}{5}$$

(1)

$$\left(\sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2\right) + \left(\sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2\right)$$

(2)

$$\Rightarrow \left(\sqrt{a} + \frac{1}{\sqrt{a}} + 2\right) + \left(\sqrt{a'} + \frac{1}{\sqrt{a'}} + 2\right) \rightarrow 2\sqrt{2-\sqrt{3}} + \frac{2}{\sqrt{2-\sqrt{3}}} + 4$$

$$\rightarrow 2\left(\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}}\right) + 4 \Rightarrow \left(\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}}\right)^2 = 2 - \sqrt{3} + 2 + 2\sqrt{3} = 4$$

$$\Rightarrow 2(\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}}) + 4 \rightarrow 2(\sqrt{4}) + 4 = 2(2) + 4 = 8$$

$$2f(-x) - 3f(-x) = 4(-x)^2 - (-x) \Rightarrow 2f(-x) - 3f(x) = 4x^2 + x \quad (3)$$

$$2\left(\frac{1}{x^2}f(x) - \frac{1}{x^2}f(-x)\right) = 4x^2 + x$$

$$+ 3\left(\frac{1}{x^2}f(-x) - \frac{1}{x^2}f(x)\right) = 4x^2 + x$$

$$\rightarrow -\frac{1}{x^2}f(x) = 4x^2 + x \rightarrow f(x) = \frac{4x^2 + x}{-1} = -4x^2 - \frac{1}{2}x$$

(4)

$$x=0 \rightarrow 2f(0) - 0 = 0 - 0 + 3m - 1 \rightarrow f(0) = \frac{3m-1}{2}$$

$$x=-2 \rightarrow 0 + 3f(0) = 16 + 2m + 3m - 1 \rightarrow f(0) = \frac{5m+15}{5}$$

$$11m - 9 = 10m + 30 \rightarrow 11m = 39 \rightarrow m = \frac{39}{11} \rightarrow f(0) = \frac{\frac{39}{11} - 1}{2} = \frac{\frac{28}{11}}{2} = \frac{14}{11}$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 - 12x + 3}{x} \rightarrow ax + \frac{a}{x} + 2b \Rightarrow a=3$$

$$b=-6$$

(5)

$$f(-1) = 3(-1) - 6 = -9$$

$$\begin{cases} f(x) \rightarrow 1-x \geq 0 \rightarrow x \leq 1 \rightarrow D_f = (-\infty, 1] \\ g(x) \rightarrow (1-x)^3 \geq 0 \rightarrow 1-x \geq 0 \rightarrow D_g = (-\infty, 1] \end{cases}$$

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$$\Rightarrow D_f = D_g \xrightarrow{\text{نظارت}} -\sqrt{(1-x)^3(1-x)} = -|1-x|\sqrt{1-x} = x-1\sqrt{1-x}$$

$$\Rightarrow f(x) = g(x) \text{ برابرند}$$

$$\begin{cases} f(x) \rightarrow x \geq 0 \rightarrow D_f = [0, +\infty) \\ g(x) \rightarrow x^3 \geq 0 \rightarrow x \geq 0 \rightarrow D_g = [0, +\infty) \end{cases}$$

(١٠)

$$\rightarrow g(x) = \sqrt{x^3 \cdot x} = |x|\sqrt{x} \rightarrow g(x) = f(x) \checkmark$$

$$\begin{cases} f(x) \rightarrow x \geq 0 \rightarrow [a - [a]] \rightarrow \underbrace{a - [a]}_0 < 1 \rightarrow \mathbb{R} \Rightarrow D_f = [0, +\infty) \\ g(x) \rightarrow x - |x| \geq 0 \rightarrow x \geq |x| \rightarrow x \geq 0 \rightarrow D_g = [0, +\infty) \end{cases} D_f = D_g$$

$$g(x) = \sqrt{x-x} \Rightarrow g(x) = 0$$

$$\rightarrow x \geq 0 \rightarrow f(x) = [\sqrt{x} - [\sqrt{x}]] \quad \boxed{f(x) = g(x)} \checkmark$$

$$[\sqrt{x}] = \mathbb{Z} \rightarrow f(x) = [\sqrt{x} - \mathbb{Z}] \rightarrow f(x) = \mathbb{Z} - \mathbb{Z} \xrightarrow{\text{ازای}} x \geq 0$$

$$\left(\frac{f+g}{f-g}\right)(2) \rightarrow \frac{f(2)+g(2)}{f(2)-g(2)} = \frac{2+2}{2-2} = \frac{4}{0} \rightarrow \text{تقسیم بر صفر} \quad \left(\frac{f+g}{f-g}\right)(1) \rightarrow \frac{f(1)+g(1)}{f(1)-g(1)} = \frac{1+1}{1-1} = \frac{2}{0} \rightarrow \text{تقسیم بر صفر}$$

$$f(0) \cdot g(-1) = 0 \cdot 1 = 0 \quad \text{ج}$$

$$g(x) = f(3x-1) \rightarrow 3x-1=2 \rightarrow \boxed{x=1} \quad | \quad 3x-1=-1 \rightarrow \boxed{x=0}$$

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$$3x-1=0 \rightarrow \boxed{x=\frac{1}{3}} \quad | \quad 3x-1=2 \rightarrow \boxed{x=\frac{3}{2}} \quad | \quad 3x-1=3 \rightarrow \boxed{x=\frac{4}{3}}$$

$$g(x) = \left\{ (1,1) (0,2) \left(\frac{1}{3},4\right) \left(-\frac{1}{3},2\right) \left(\frac{2}{3},-1\right) \right\}$$

$$f(x) = f(|x|) \quad \text{ب}$$

$$|x|=2 \rightarrow x=\pm 2$$

$$f(x) = \left\{ (2,1) (-2,1) (0,2) (2,-1) (-2,-1) \right\} \quad |x|=1 \rightarrow x=\pm 1 \quad |x|=0 \rightarrow x=0$$

$$|x|=3 \rightarrow x=\pm 3$$