

$$2f(a) = f(-2) + a \quad f(x) = \begin{cases} (a+1)(x+2) & x > 1 \\ 3a + 2x & x \leq 1 \end{cases}$$

$$x = a \rightarrow f(x) = 7a + 7$$

$$x = -2 \rightarrow f(x) = 3a - 2$$

$$\rightarrow 12a + 12 = 3a - 2 \Rightarrow 10a = -14 \Rightarrow \boxed{a = -1.4}$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \quad f(2-\sqrt{3}) + f(2+\sqrt{3}) = ?$$

$$x = 2 - \sqrt{3} \Rightarrow \frac{\sqrt{2-\sqrt{3}}}{1 \times \sqrt{2-\sqrt{3}}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2 \Rightarrow \frac{2-\sqrt{3}+1}{\sqrt{2-\sqrt{3}}} + 2 = \frac{3-\sqrt{3}}{\sqrt{2-\sqrt{3}}} + 2$$

$$x = 2 + \sqrt{3} \Rightarrow \frac{\sqrt{2+\sqrt{3}}}{1 \times \sqrt{2+\sqrt{3}}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2 \Rightarrow \frac{2+\sqrt{3}+1}{\sqrt{2+\sqrt{3}}} + 2 = \frac{3+\sqrt{3}}{\sqrt{2+\sqrt{3}}} + 2$$

$$f(2-\sqrt{3}) + f(2+\sqrt{3}) = \frac{3-\sqrt{3}}{\sqrt{2-\sqrt{3}}} + \frac{3+\sqrt{3}}{\sqrt{2+\sqrt{3}}} + 4 \Rightarrow (3-\sqrt{3})(\sqrt{2+\sqrt{3}}) + (3+\sqrt{3})(\sqrt{2-\sqrt{3}}) + 4$$

$$\begin{aligned} x = x & \begin{cases} 2(2f(x) - 2f(-x)) = 2x^2 - x \\ 3(2f(-x) - 3f(x)) = 2x^2 + x \end{cases} \\ x = -x & \end{aligned}$$

$$\begin{aligned} & \begin{cases} 2f(x) - 2f(-x) = x^2 - 2x \\ 2f(-x) - 3f(x) = x^2 + 3x \end{cases} \Rightarrow -\Delta f(x) = 5x^2 + x \\ & \begin{cases} 2f(x) - 2f(-x) = x^2 - 2x \\ 2f(-x) - 3f(x) = x^2 + 3x \end{cases} \end{aligned}$$

$$f(x) = \frac{5x^2 + x}{-5} = \boxed{-x^2 - \frac{1}{5}x}$$

$$(x+2)f(x) - 3xf(x+2) = 2x^2 - mx + 3m - 1$$

$$x = 0 \rightarrow 2f(0) = 3m - 1 \Rightarrow f(0) = \frac{3m-1}{2}$$

$$x = -2 \rightarrow 4f(0) = 12 + 2m \Rightarrow f(0) = \frac{12+2m}{4}$$

$$\frac{12+2m}{4} = \frac{3m-1}{2} \quad 12+2m = 6m-2 \Rightarrow m = 4$$

$$f(0) = \frac{(3 \times 4) - 1}{2} = \boxed{5.5}$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 + 12x + 3}{x} \quad \text{خطی است}$$

$$x = -1 \Rightarrow 2f(-1) = -18 \Rightarrow f(-1) = \boxed{-9}$$

$f(u) = (u-1)\sqrt{1-u} \Rightarrow D_f = (-\infty, 1] \checkmark \rightarrow R_f = (-\infty, 0] \checkmark$
 $g(u) = -\sqrt{(1-u)^3} \Rightarrow D_g = (-\infty, 1] \rightarrow R_g = (-\infty, 0]$
 $1-u \geq 0$
 $(u-1)\sqrt{1-u} = -\sqrt{(1-u)^3} \Rightarrow u-1 = -\sqrt{(1-u)^2} \Rightarrow u-1 = -(1-u) \checkmark$
 $D = (-\infty, 1]$

تساوی برقرار است

$f(u) = u\sqrt{u} \Rightarrow D_f = [0, +\infty) \checkmark \rightarrow R_f = [0, +\infty) \checkmark$
 $g(u) = \sqrt{u^3} \Rightarrow D_g = [0, +\infty) \rightarrow R_g = [0, +\infty) \checkmark$
 $u\sqrt{u} = \sqrt{u^3} \Rightarrow u = \sqrt{\frac{u^3}{u}} \Rightarrow u = \sqrt{u^2} \Rightarrow u = |u| \checkmark$
 $D = [0, +\infty)$

تساوی برقرار است

$f(u) = [\sqrt{u} - \sqrt{u}] \Rightarrow D = [0, +\infty) \rightarrow R_f = \{0\} \checkmark$
 $g(u) = \sqrt{u-|u|} \Rightarrow D = [0, +\infty) \rightarrow R_g = \{0\} \checkmark$

$u = x^2 \Rightarrow [\sqrt{x} - \sqrt{x}] = 0 \checkmark$
 $u = \frac{1}{x} \Rightarrow [\sqrt{\frac{1}{x}} - \sqrt{\frac{1}{x}}] = 0 \checkmark$

تساوی برقرار است

$f = \{(1, 2)(2, 3)(3, 1)(0, 0)\} \quad g = \{(-1, 1)(1, 1)(2, 3)(3, 5)(0, 2)(\sqrt{0}, 1)\}$
 الف) $(\frac{f+g}{f-g})(1) \Rightarrow \{(1, \frac{3}{1}) \text{ (highlighted)} (2, \frac{4}{-1}) (0, \frac{-2}{-1})\} \Rightarrow \{(1, 3) (2, -4) (0, 2)\}$
 ب) $(\frac{f+g}{f-g})(2) \Rightarrow \{(1, \frac{4}{-1})\} \Rightarrow \{(1, -4)\}$

ج) $f(0) \text{ و } g(1) \Rightarrow 0$
 $\frac{f(x)+g(x)}{f(x)-g(x)} = \frac{3+3}{3-3} = \frac{6}{0} = \text{ت.ج.}$

1, 1/2

$f(u) = \{(2, 1)(-1, 2)(0, 5)(-2, 2)(3, -1)\}$

الف) $g(u) = f(xu-1) \Rightarrow \{(1, 1)(0, 2)(\frac{1}{x}, 5)(-\frac{1}{x}, 2)(\frac{x}{x}, -1)\}$

$xu-1=2 \Rightarrow u=1 \rightarrow g(1)=f(2)$
 $xu-1=0 \Rightarrow u=\frac{1}{x} \rightarrow g(\frac{1}{x})=f(1)$
 $xu-1=-1 \Rightarrow u=0 \rightarrow g(0)=f(-1)$
 $xu-1=-2 \Rightarrow u=-\frac{1}{x} \rightarrow g(-\frac{1}{x})=f(-2)$

$xu-1=3 \Rightarrow u=\frac{4}{x} \rightarrow g(\frac{4}{x})=f(3)$

ب) $h(u) = f(|u|)$

$|u|=2 \Rightarrow u=\pm 2$

$h(2) = f(2) / h(-2) = f(2)$

$|u|=0 \Rightarrow u=0$

$h(0) = f(0)$

$|u|=3 \Rightarrow u=\pm 3$

$h(3) = f(3)$

$h(-3) = f(3)$

$\{(2, 1)(-2, 1)(0, 5)(\frac{4}{x}, -1)(-\frac{4}{x}, -1)\}$

$$f(2-\sqrt{3}) = \sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2 \quad (2)$$

$$\xrightarrow{(2-\sqrt{3})(2+\sqrt{3})=1} \frac{1}{\sqrt{2-\sqrt{3}}} = \sqrt{2+\sqrt{3}}$$

$$f(2+\sqrt{3}) = \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2$$

$$f(2-\sqrt{3}) = \sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}} + 2$$

$$و \quad f(2+\sqrt{3}) = \sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}} + 2$$

اگر $A = \sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}}$ ارتفاع بگیریم، داریم:

$$A^2 = 2+\sqrt{3} + 2-\sqrt{3} + 2\sqrt{(2-\sqrt{3})(2+\sqrt{3})} = 4 \rightarrow A = \sqrt{4}$$

$$f(2+\sqrt{3}) + f(2-\sqrt{3}) = \sqrt{4} + 2 + \sqrt{4} + 2 = \boxed{2\sqrt{4} + 4}$$

البته جواب $(2(\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}} + 2))$ هم درست است ولی این جواب زیاده ت

$$\text{الف) } \frac{f(1)+g(1)}{f(1)-g(1)} = \frac{2+1}{2-1} = 3$$

(9)

$$\text{ب) } \frac{f(2)+g(2)}{f(2)-g(2)} = \frac{3+3}{3-3} = \frac{6}{0} \text{ تعریف نشده!}$$

$$\text{پ) } f(0) \times g(-1) = 0 \times 1 = 0$$