

$$2f(a) = f(-2) + a \quad f(x) = \begin{cases} (a+1)(x+2) & x > 1 \\ 3a + 2x & x \leq 1 \end{cases}$$

$$x = a \rightarrow f(x) = 7a + 7$$

$$x = -2 \rightarrow f(x) = 3a - 2$$

$$\rightarrow 12a + 12 = 3a - 2 \Rightarrow 10a = -14 \Rightarrow \boxed{a = -1,4}$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \quad f(2-\sqrt{3}) + f(2+\sqrt{3}) ?$$

$$x = 2 - \sqrt{3} \Rightarrow \frac{\sqrt{2-\sqrt{3}}}{1 \times \sqrt{2-\sqrt{3}}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2 \Rightarrow \frac{2-\sqrt{3}+1}{\sqrt{2-\sqrt{3}}} + 2 = \frac{3-\sqrt{3}}{\sqrt{2-\sqrt{3}}} + 2$$

$$x = 2 + \sqrt{3} \Rightarrow \frac{\sqrt{2+\sqrt{3}}}{1 \times \sqrt{2+\sqrt{3}}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2 \Rightarrow \frac{2+\sqrt{3}+1}{\sqrt{2+\sqrt{3}}} + 2 = \frac{3+\sqrt{3}}{\sqrt{2+\sqrt{3}}} + 2$$

$$f(2-\sqrt{3}) + f(2+\sqrt{3}) = \frac{3-\sqrt{3}}{\sqrt{2-\sqrt{3}}} + \frac{3+\sqrt{3}}{\sqrt{2+\sqrt{3}}} + 2 \Rightarrow \boxed{(3-\sqrt{3})(\sqrt{2-\sqrt{3}}) + (3+\sqrt{3})(\sqrt{2+\sqrt{3}}) + 2}$$

$$\begin{aligned} x = x & \begin{cases} 2(f(x) - f(-x)) = \varepsilon x^2 - x \\ 3(f(x) - f(-x)) = \varepsilon x^2 + x \end{cases} \\ x = -x & \begin{cases} 2(f(x) - f(-x)) = \varepsilon x^2 - x \\ 3(f(x) - f(-x)) = \varepsilon x^2 + x \end{cases} \end{aligned}$$

$$+ \begin{cases} \varepsilon f(x) - 4f(-x) = 12x^2 - 2x \\ 4f(-x) - 9f(x) = 12x^2 + 3x \end{cases} \Rightarrow -5f(x) = 10x^2 + x$$

$$f(x) = \frac{10x^2 + x}{-5} = \boxed{-2x^2 - \frac{1}{5}x}$$

$$(x+2)f(x) - 3xf(x+2) = \varepsilon x^2 - mx + 3m - 1$$

$$x=0 \rightarrow 2f(0) = 3m - 1 \Rightarrow f(0) = \frac{3m-1}{2}$$

$$x=-2 \rightarrow 4f(0) = 12 + 2m \Rightarrow f(0) = \frac{12+2m}{4}$$

$$\frac{12+2m}{4} = \frac{3m-1}{2} \quad 12+2m = 6m-2 \Rightarrow m = 2, 4$$

$$f(0) = \frac{(3 \times 4) - 1}{2} = \boxed{4, 2, 4}$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 + 12x + 3}{x} \quad \text{خطی است}$$

$$x=-1 \Rightarrow 2f(-1) = -14 \Rightarrow f(-1) = \boxed{-7}$$

$$f(u) = (u-1)\sqrt{1-u} \Rightarrow D_f = (-\infty, 1] \quad \checkmark \rightarrow R_f = (-\infty, 0] \quad \checkmark$$

$$g(u) = -\sqrt{(1-u)^3} \Rightarrow D_g = (-\infty, 1] \rightarrow R_g = (-\infty, 0]$$

$$(u-1)\sqrt{1-u} = -\sqrt{(1-u)^3} \quad u-1 = -\sqrt{(1-u)^2} \Rightarrow u-1 = -(1-u) \quad \checkmark$$

$$D = (-\infty, 1]$$

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$$f(u) = u\sqrt{u} \Rightarrow D_f = [0, +\infty) \quad \checkmark \rightarrow R_f = [0, +\infty) \quad \checkmark$$

$$g(u) = \sqrt{u^3} \Rightarrow D_g = [0, +\infty) \rightarrow R_g = [0, +\infty) \quad \checkmark$$

$$u\sqrt{u} = \sqrt{u^3} \quad u = \sqrt{\frac{u^3}{u}} \Rightarrow u = \sqrt{u^2} \Rightarrow u = |u| \quad \checkmark$$

$$D = [0, +\infty)$$

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$$f(u) = [\sqrt{u} - \sqrt{u}] \Rightarrow D = [0, +\infty) \rightarrow R_f = \{0\}$$

$$g(u) = \sqrt{u-|u|} \Rightarrow D = [0, +\infty) \rightarrow R_g = \{0\}$$

$$u = x^2 \Rightarrow [\sqrt{x^2} - \sqrt{x^2}] = 0$$

$$u = \frac{1}{x^2} \Rightarrow [\sqrt{\frac{1}{x^2}} - \sqrt{\frac{1}{x^2}}] = 0$$

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$$f = \{(1, 2)(2, 3)(3, 1)(0, 0)\} \quad g = \{(-1, 1)(1, 1)(2, 3)(3, 5)(0, 2)(\sqrt{5}, 1)\}$$

$$\text{الف) } \left(\frac{f+g}{f-g}\right)(1) \Rightarrow \{(1, \frac{3}{1})(2, \frac{4}{0})(3, \frac{4}{-2})(0, \frac{-1}{-1})\} \Rightarrow \{(1, 3)(3, -\frac{1}{2})(0, 1)\}$$

$$\text{ب) } \left(\frac{f+g}{f-g}\right)(2) \Rightarrow \{(1, \frac{4}{-2})\} \Rightarrow \{(1, -\frac{1}{2})\}$$

$$\text{ج) } f(0) \cdot g(1) \Rightarrow 0$$

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$$f(u) = \{(2, 1)(-1, 2)(0, 5)(-2, 2)(3, -1)\}$$

$$\text{الف) } g(u) = f(u-1) \Rightarrow \{(1, 1)(0, 2)(\frac{1}{2}, 5)(-\frac{1}{2}, 2)(\frac{3}{2}, -1)\}$$

$$u-1 = 2 \Rightarrow u = 3 \rightarrow g(3) = f(2) \quad u-1 = 0 \Rightarrow u = 1 \rightarrow g(1) = f(0) \quad u-1 = -1 \Rightarrow u = 0 \rightarrow g(0) = f(-1)$$

$$u-1 = 3 \Rightarrow u = \frac{4}{2} \rightarrow g(\frac{4}{2}) = f(3)$$

$$\text{ب) } h(u) = f(|u|)$$

$$|u| = 2 \Rightarrow u = \pm 2$$

$$h(2) = f(2) / h(-2) = f(2)$$

$$|u| = 0 \Rightarrow u = 0$$

$$h(0) = f(0)$$

$$\{(2, 1)(-2, 1)(0, 5)(3, -1)(-3, -1)\}$$

$$|u| = 3 \Rightarrow u = \pm 3$$

$$h(3) = f(3)$$

$$h(-3) = f(3)$$

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