

$$f(x) = \begin{cases} (a+1)(x+2) & ; x > 1 \\ 3a+2x & ; x \leq 1 \end{cases} \quad \begin{aligned} 2f(5) &= f(-2) + a \rightarrow 2(a+1)(7) = (3a-2) + a \\ &\rightarrow 14a+14 = 4a-2 \rightarrow 10a = -16 \rightarrow a = \frac{-16}{10} = \frac{-8}{5} = -1,6 \end{aligned} \quad (1)$$

$$\begin{aligned} f(x) &= \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \\ f(2-\sqrt{3}) + f(2+\sqrt{3}) &\rightarrow \left(\sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2\right) + \left(\sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2\right) \rightarrow \left(\sqrt{2} + \frac{1}{\sqrt{2}} + 2\right) + \left(\frac{1}{\sqrt{2}} + \sqrt{2} + 2\right) \Rightarrow \\ &\quad \sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2 + \frac{1}{\sqrt{2+\sqrt{3}}} + \sqrt{2+\sqrt{3}} + 2 \rightarrow 2\sqrt{2-\sqrt{3}} + \frac{2}{\sqrt{2-\sqrt{3}}} + 4 \\ &\rightarrow 2(\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}}) + 4 \rightarrow (\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}})^2 = 2 - \sqrt{3} + 2 + \sqrt{3} = 4 \\ &\rightarrow 2(\sqrt{4}) + 4 \rightarrow f(2-\sqrt{3}) + f(2+\sqrt{3}) = 4 + 2\sqrt{4} \end{aligned} \quad (2)$$

$$\begin{aligned} 2f(x) - 3f(-x) &= \varepsilon x^2 - x \\ f(x) &=? \\ &\rightarrow 2f(-x) - 3f(-(-x)) = \varepsilon(-x)^2 - (-x) = 2f(-x) - 3f(x) = \varepsilon x^2 + x \\ &\Rightarrow \begin{cases} 2f(x) - 3f(-x) = \varepsilon x^2 - x \\ 2f(-x) - 3f(x) = \varepsilon x^2 + x \end{cases} \rightarrow \begin{aligned} &\varepsilon f(x) - 4f(-x) = 18x^2 - 2x \\ &4f(-x) - 9f(x) = 12x^2 + 3x \end{aligned} \\ &\rightarrow -5f(x) = 10x^2 + x \rightarrow f(x) = \frac{10x^2 + x}{-5} \rightarrow f(x) = -2x^2 - \frac{1}{5}x \end{aligned} \quad (3)$$

$$\begin{aligned} (x+2)f(x) - 3xf(x+2) &= \varepsilon x^2 - mx + 3m - 1 \\ f(0) &=? \quad \begin{aligned} \text{if } x=0 &\rightarrow 2f(0) - 0 = 0 - 0 + 3m - 1 \rightarrow 2f(0) = 3m - 1 \rightarrow f(0) = \frac{3m-1}{2} \\ \text{if } x=-2 &\rightarrow 0 + 4f(0) = 4 + 2m + 3m - 1 \rightarrow 4f(0) = 5m + 3 \rightarrow f(0) = \frac{5m+3}{4} \\ &\rightarrow \frac{3m-1}{2} = \frac{5m+3}{4} \rightarrow 11m - 4 = 10m + 3 \rightarrow 1m = 7 \rightarrow m = 7 \\ &\rightarrow f(0) = \frac{3(7)-1}{2} = \frac{20}{2} = 10 \end{aligned} \end{aligned} \quad (4)$$

$$\begin{aligned} f(x) &= ax + b \\ f(x) + f\left(\frac{1}{x}\right) &= \frac{3x^2 - 12x + 3}{x} \rightarrow ax + b + \frac{a}{x} + b = \frac{3x^2 - 12x + 3}{x} \rightarrow ax + \frac{a}{x} + 2b = 3x - 12 + \frac{3}{x} \\ &\rightarrow a = 3, b = -9 \rightarrow f(-1) = 3(-1) - 9 \rightarrow f(-1) = -12 \end{aligned} \quad (5)$$

$$\begin{aligned} f(x) &= (x-1)\sqrt{1-x} \rightarrow 1-x \geq 0 \rightarrow x \leq 1 \rightarrow D_f = (-\infty, 1] \\ g(x) &= -\sqrt{1-(x-1)^2} \rightarrow (1-x)^2 \geq 0 \rightarrow 1-x \geq 0 \rightarrow x \leq 1 \rightarrow D_g = (-\infty, 1] \\ &\rightarrow D_f = D_g \checkmark \\ &\text{نسای برقرار است.} \\ &\text{با توجه به دامنه تغییرات مثبت} \rightarrow g(x) = (x-1)\sqrt{1-x} \rightarrow f(x) = g(x) \checkmark \end{aligned} \quad (6)$$

$$\begin{cases} f(x) = x\sqrt{x} \rightarrow x \geq 0 \rightarrow D_f = [0, +\infty) \\ g(x) = \sqrt{x^3} \rightarrow x^3 \geq 0 \rightarrow x \geq 0 \rightarrow D_g = [0, +\infty) \end{cases} \rightarrow D_f = D_g \checkmark$$

تساوی برقرار است.

(۷)

$$\xrightarrow{\text{برای ضابطه}} g(x) = \sqrt{x^3 \cdot x} = |x| \sqrt{x} \xrightarrow{\text{با توجه به شرط } x \geq 0} g(x) = x\sqrt{x} \rightarrow f(x) = g(x) \checkmark$$

$$\begin{cases} f(x) = [\sqrt{x} - [\sqrt{x}]] \rightarrow x \geq 0 \rightarrow [a - [a]] \rightarrow 0 \leq a - [a] < 1 \rightarrow [a - [a]] = 0 \rightarrow D = \mathbb{R} \rightarrow D_f = [0, +\infty) \cap (-\infty, +\infty) \cap \mathbb{Z} \rightarrow D_f = [0, +\infty) \\ g(x) = \sqrt{x - |x|} \rightarrow x - |x| \geq 0 \rightarrow x \geq |x| \rightarrow x \geq 0 \rightarrow D_g = [0, +\infty) \end{cases} \rightarrow D_f = D_g \checkmark$$

$$\xrightarrow{\text{برای ضابطه}} g(x) = \sqrt{x - x} \rightarrow g(x) = 0 \text{ برای } x \geq 0 \text{ و } f(x) = [\sqrt{x} - [\sqrt{x}]] \xrightarrow{[\sqrt{x}] = \mathbb{Z}} f(x) = [\sqrt{x} - \mathbb{Z}]$$

$$\rightarrow f(x) = [\sqrt{x}] - \mathbb{Z} \rightarrow f(x) = \mathbb{Z} - \mathbb{Z} \rightarrow f(x) = 0 \text{ و } x \geq 0$$

$$\rightarrow f(x) = g(x) \checkmark$$

تساوی برقرار است.

$$f = \{(1, 2), (2, 3), (3, 1), (0, 0)\}$$

(۹)

$$g = \{(-1, 1), (1, 1), (2, 3), (3, 5), (0, 2), (\sqrt{2}, 1)\}$$

$$\text{الف) } \left(\frac{f+g}{f-g} \right) (1) \rightarrow \frac{f(1)+g(1)}{f(1)-g(1)} = \frac{2+1}{2-1} = 3$$

$$\text{ب) } \left(\frac{f+g}{f-g} \right) (2) \rightarrow \frac{f(2)+g(2)}{f(2)-g(2)} = \frac{3+3}{3-3} = \frac{6}{0} \rightarrow \text{تعریف نشده}$$

$$\text{ج) } f(0)g(-1) = 0 \times 1 = 0$$

$$f(x) = \{(2, 1), (-1, 2), (0, 5), (-2, 2), (3, -1)\}$$

(۱۰)

$$\text{الف) } g(x) = f(3x-1) \rightarrow 3x-1=2 \rightarrow 3x=3 \rightarrow x=1 \text{ و } 3x-1=-1 \rightarrow 3x=0 \rightarrow x=0 \text{ و } 3x-1=0 \rightarrow 3x=1 \rightarrow x=\frac{1}{3} \text{ و } 3x-1=-2 \rightarrow 3x=-1 \rightarrow x=-\frac{1}{3} \text{ و } 3x-1=3 \rightarrow 3x=4 \rightarrow x=\frac{4}{3} \rightarrow g(x) = \{(1, 1), (0, 2), (\frac{1}{3}, 5), (-\frac{1}{3}, 2), (\frac{4}{3}, -1)\}$$

$$\text{ب) } h(x) = f(|x|) \rightarrow |x|=2 \rightarrow x=2 \text{ یا } x=-2 \text{ و } |x|=1 \text{ و غیرممکن } |x|=0 \rightarrow x=0 \text{ و } |x|=-2 \text{ و غیرممکن } |x|=3 \rightarrow x=3 \text{ یا } x=-3 \rightarrow h(x) = \{(2, 1), (-2, 1), (0, 5), (3, -1), (-3, -1)\}$$