

۱. آزمون علامت در مشتق

$f(x) = \sqrt{x+4} - \sqrt{x-4}$ و $f(x) + g(x) = a + b\sqrt{x+c}$

$g(x) = \sqrt{x-4} - \sqrt{x+4}$ $x > 4$ $\frac{a+b}{c} = ?$ $\left(-\frac{1}{4}\right)$

$t = \sqrt{x-4} \rightarrow x = t^2 + 4$
 $f(t) = \sqrt{t^2+4+4} - \sqrt{t^2+4-4} = \sqrt{t^2+8} - t$
 $g(t) = \sqrt{t^2+4-4} - \sqrt{t^2+4+4} = t - \sqrt{t^2+8}$
 $f(t) + g(t) = \sqrt{t^2+8} - t + t - \sqrt{t^2+8} = 0$
 $\frac{a+b}{c} = \frac{0}{-4} = 0$ $a=0, b=2, c=-4$

$f(x) = \frac{x+2}{x^2-4x+3}$ $f(0) = \frac{2}{3}$
 $g(x) = \{ (2,1), (1,0), (3,2), (0,4) \}$ $g(2) = -4$

$x^2 - 4x + 3 = 0$ $x = 2, 1$
 $g(x) = \{ (2,1), (1,0), (3,2), (0,4) \}$
 $g(2) = 1$ $g(1) = 0$ $g(3) = 2$ $g(0) = 4$

$\frac{g}{f} = \{ (0,4), (2,1) \}$

$f(x) = \{ (1,2), (3,-1), (4,2), (-1,9) \}$
 $g(x) = \{ (-2,3), (1,-1), (3,2), (-1,0) \}$

$f(x) = \{ (1,2), (3,-1), (4,2), (-1,9) \}$
 $g(x) = \{ (-2,3), (1,-1), (3,2), (-1,0) \}$
 $h(x) = f(x) + g(x) = \{ (-1,11), (4,1), (7,4), (0,9) \}$

$g(-1) = 3$ $g(1) = -1$
 $g(3) = 2$ $g(-1) = 0$
 $2(3) + 1 = 7$ $2(1) + (-1) = 1$
 $2(-1) + 3 = 1$ $2(0) + 0 = 0$

$f(x) = \{ (2,1), (3,2), (4,9), (-2,0) \}$
 $g(x) = \{ (2,1), (3,2), (-2,9), (-1,2), (3,3) \}$
 $h = f + g = ?$
 $h = \{ (2,2), (3,4), (-2,9) \}$

$h(x) = f(x) + g(x)$
 $x = 2 \rightarrow h(2) = 2 + 1 = 3$
 $x = 3 \rightarrow h(3) = 2 + 2 = 4$
 $x = -2 \rightarrow h(-2) = 0 + 9 = 9$

$f(x) = \frac{\sqrt{3x-x}}{x+1}$
 $g(x) = [x] + [-x]$
 $\lim_{x \rightarrow 0} \left(\frac{f}{g+1} \right)(x) = ?$

$D = \{ 2, 3 \}$ $f = \{ 2, 3 \}$

$f(x) = \sqrt{3x-x} = \sqrt{2x}$ $x > 0$
 $g(x) = [x] + [-x] = -1$ $x \in \mathbb{Z}$
 $\lim_{x \rightarrow 0} \left(\frac{f}{g+1} \right)(x) = \lim_{x \rightarrow 0} \left(\frac{\sqrt{2x}}{-1+1} \right) = \lim_{x \rightarrow 0} \left(\frac{\sqrt{2x}}{0} \right) = \infty$

$g(x) + 1 = 0$ $[x] + [-x] = -1$ $x \in \mathbb{Z}$
 $g(x) = -1$ $g(x) + 1 = 0$

$f(x) = \sqrt{2x - x^2}$ $\rightarrow x - x^2 \geq 0 \rightarrow x^2 - 2x \leq 0$ $(x-2)(x-0) \leq 0$ $g(x) = \{(1, 2), (2, 1), (4, 4), (5, 1)\}$ $h(x) = \frac{f(x)}{g(x)}$ $Lh = \{(1, 0), (2, \frac{\sqrt{2}}{2}), (4, 0)\}$	$x=1, 2, 4$ $n=1 \rightarrow f(1) = \sqrt{2-1} = 1 \rightarrow h(1) = \frac{1}{2}$ $n=2 \rightarrow f(2) = \sqrt{4-4} = 0 \rightarrow h(2) = \frac{0}{1} = 0$ $n=4 \rightarrow f(4) = \sqrt{8-16} = 0 \rightarrow h(4) = 0$
$y = x^2 - 2x^2 + 3x = (x-1)^2 + 1$ $y = 1 = (x-1)^2 \rightarrow x-1 = \sqrt{y-1} \rightarrow x = \sqrt{y-1} + 1$ $R_y = [1, \infty)$ $y = \frac{1}{x^2 - 2x} \rightarrow yx^2 - 2yx = 1 \rightarrow yx^2 - 2yx - 1 = 0$ $\Delta = (-2y)^2 - 4y(-1) = 4y^2 + 4y = 4y(y+1)$ $y > 0 \rightarrow y+1 > 0 \rightarrow y > -1$ $R_y = (-\infty, -1] \cup (0, +\infty)$	$y = (x-1)^2 + 1$ $y = 1$ $y = \frac{1}{x^2 - 2x}$
$y = x^2 - 4x + 6 \rightarrow (x-2)^2 + 2$ $R_y = [2, +\infty)$ $y = -x^2 + 4x + 5 \rightarrow -(x-2)^2 + 9$ $R_y = (-\infty, 9]$ $y = \sqrt{x^2 - 4x} \rightarrow \sqrt{(x-2)^2 - 4}$ $R_y = [2, +\infty)$ $y = \sqrt{4x - x^2} \rightarrow \sqrt{4 - (x-2)^2}$ $R_y = [0, 2]$	$(\frac{1}{2}, 4) = (2, 4)$ $(-\frac{9}{2}, 12) = (4, 12)$ $(\frac{1}{2}, -1)$ $(-\frac{9}{2}, 9)$
$y = x^2 - 2x^2 + 2x + 1 = -x^2 + 2x + 1$ $R_y = \mathbb{R}$ $y = x^2 - 4x^2 + 3x + 2 = -3x^2 + 3x + 2$ $R_y = \mathbb{R}$ $y = \sqrt{x^2 - 4x + 4} = x-2$ $R_y = [0, +\infty)$ $y = (x^2 - 4x^2 + 3x + 1)^2$ $R_y = [0, +\infty)$	$R_y = [2, 4]$ $R_y = \mathbb{R}$ $R_y = [0, +\infty)$ $R_y = [0, +\infty)$
$y = \frac{2x+1}{x-2} \rightarrow R_y = \mathbb{R} - \{\frac{a}{c} = 2\} = \mathbb{R} - \{2\}$ $y = \frac{2x+1}{x+1} \rightarrow R_y = \mathbb{R} - \{\frac{a}{c} = \frac{2}{1} = 2\} = \mathbb{R} - \{2\}$	$\frac{a}{c} = 2$ $\frac{a}{c} = \frac{2}{1}$