

$$(f(n) + g(n))^2 = f^2 + g^2 + 2fg \rightarrow f^2 = n + 4\sqrt{n-4} / g^2 = n - 4\sqrt{n-4}$$

$$f^2 + g^2 = 2n$$

$$f \times g = \sqrt{(n+4\sqrt{n-4})(n-4\sqrt{n-4})} = \sqrt{n^2 - 16n + 16} = \sqrt{(n-4)^2}, n \geq 4$$

$$(f+g)^2 = 2n + 2(n-4) = 4(n-2) \rightarrow f+g = 2\sqrt{n-2} = a+b\sqrt{n+c}$$

$$a+b = 2\sqrt{n-2} = a+b\sqrt{n+c} \rightarrow \begin{cases} a=0 \\ b=2 \\ c=-4 \end{cases} \Leftrightarrow |n-1| = n-1$$

$$\frac{a+b}{c} = \frac{0+2}{-4} = -\frac{1}{2}$$

باید مقدار $f(n)$ را برای n های خاص حساب کنیم.

$$f(2) = -4, g(2) = 1 \rightarrow \frac{g(2)}{f(2)} = -\frac{1}{4}$$

$$f(1) = \frac{3}{0} \rightarrow \text{تعریف نشده} \times \quad f(0) = \frac{2}{4} \left\{ \frac{g(0)}{f(0)} = 4 \times \frac{3}{4} = 3 \right\}$$

$$f(3) = \frac{5}{0} \rightarrow \text{تعریف نشده} \times \quad g(0) = 4 \left\{ \frac{g(0)}{f(0)} = 4 \times \frac{3}{4} = 3 \right\}$$

$$n \in \{1, 3, 4, -1\} \rightarrow n \in \{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, -\frac{1}{2}\} \rightarrow f(n) = \left\{ \left(\frac{1}{2}, 2\right), \left(\frac{3}{2}, -1\right), \left(\frac{5}{2}, 2\right), \left(-\frac{1}{2}, 2\right) \right\}$$

$$n^2 \in \{1, 3, 4, \dots\} \rightarrow n^2 = 1 \rightarrow n = \pm 1$$

$$n^2 = 3 \rightarrow n = \pm\sqrt{3} \quad f(n) = \left\{ (-2, 2), (-1, 2), (1, 2), (2, 2), (-\sqrt{3}, -1), (\sqrt{3}, -1) \right\}$$

$$n^2 = 4 \rightarrow n = \pm 2$$

$$g(-2) = 3 \rightarrow 2(2)^2 + 1 = 19$$

$$g(1) = -1 \rightarrow 2(-1)^2 + 1 = 3$$

$$g(4) = 2 \rightarrow 2(2)^2 + 1 = 9$$

$$g(-1) = 0 \rightarrow 2(0)^2 + 1 = 1$$

$$g(n) \neq 0 \quad \left\{ \frac{f(n)}{g(n)} = -4, \frac{f(2)}{g(2)} = -\frac{1}{4}, \frac{f(4)}{g(4)} = -\frac{1}{2}, \frac{f(-1)}{g(-1)} = -1 \right\}$$

$$2f/g = \{1, -4, 3, -1\} \rightarrow \text{جواب}$$

$$h = \{(2, \infty), (3, \infty), (-2, \infty)\}$$

$$D_f = n \geq 0, \sqrt{3n} - n \geq 0, n \neq 2 \rightarrow D_f = [0, 3] - \{2\}$$

$$D = \{0, 1, 3\}$$

$$g(n) = \begin{cases} 0 & n \in \mathbb{Z} \\ -1 & n \notin \mathbb{Z} \end{cases} \rightarrow \text{جواب}$$

$$\text{شماره منفرجه} = 0 \rightarrow \text{این حالت محال است.}$$

$$n=1 \rightarrow f(1)=1, g(1)=2 \rightarrow h(1)=1/2$$

$$n=2 \rightarrow f(2)=2, g(2)=2 \rightarrow h(2)=1$$

$$n=3 \rightarrow f(3)=2, g(3)=4 \rightarrow h(3)=1/2$$

$$n=4 \rightarrow f(4)=1, g(4)=1 \rightarrow h(4)=1$$

$$h = \left\{ \begin{array}{l} (1, 1/2) \\ (2, 1) \\ (3, 1/2) \\ (4, 1) \end{array} \right\}$$

$$y = n(n-1)(n-2)$$

$$n \rightarrow +\infty \Rightarrow y \rightarrow +\infty$$

$$n \rightarrow -\infty \Rightarrow y \rightarrow -\infty$$

$$R_f = \mathbb{R}$$

$$y(n^2 - 2n) = 1 \rightarrow n^2 - 2n = 1/y \rightarrow (n-1)^2 = 1/y + 1$$

$$n-1 = \sqrt{1/y + 1} \rightarrow 1/y + 1 \geq 0 \rightarrow y \geq -1 \rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$$

$$a = 1 \rightarrow y = -1/2 \rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$$

$$b = 2 \rightarrow y = -4/3 \rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$$

$$c = 3 \rightarrow y = -9/4 \rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$$

$$d = 4 \rightarrow y = -16/5 \rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$$

$$R_f = \mathbb{R}$$

الف) $R_f = \mathbb{R}$

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ب) $R_f = \mathbb{R}$

$$\sqrt{(-\infty, +\infty)} \Rightarrow R_f = [0, +\infty)$$

$$[(-\infty, +\infty)]^2 \Rightarrow R_f = [0, +\infty)$$

$$a = 2, c = 1 \rightarrow R_f = \mathbb{R} - \{1/2\} \Rightarrow R_f = \mathbb{R} - \{2\}$$

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