

$$f(x) = \sqrt{x+4} \sqrt{x-4} \quad g(x) = \sqrt{x-4} \sqrt{x-4} \quad x \geq 4 \quad f(x) + g(x) = a + b \sqrt{x+c}$$

$$(f+g)^2 = f^2 + g^2 + 2fg \rightarrow f^2, g^2 = (x+4)(x-4) + (x-4)(x-4) = 2$$

$$fg = \sqrt{(x+4)(x-4)(x-4)(x-4)} = \sqrt{x^2 - 16(x-4)} = \sqrt{x^2 - 16x + 64} = \sqrt{(x-4)^2} \xrightarrow{x \geq 4} \sqrt{(x-4)^2} = x-4 \rightarrow (f+g)^2 = 2x + 2(x-4) = 4x - 6 \Rightarrow f+g = \sqrt{4x-6} = 2\sqrt{x-4} \Rightarrow$$

$$a=0, b=2, c=-4 \quad \frac{a+ib}{c} = \frac{0+2i}{-4} = -\frac{1}{2}i$$

1

$$f(x) = \frac{x+4}{x^2-4x+4} \quad g(x) = \{(2,1), (1,0), (2,4), (0,4)\} \quad D_f = \mathbb{R} - \{1, 3\}$$

$$\frac{g}{f} = \left\{ (2, \frac{1}{-4}), (0, \frac{4}{\frac{4}{1}}) \right\} \quad f(2) = \frac{4}{4-8+4} = -1 \quad f(0) = \frac{4}{0-0+4} = 1$$

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$$f(x) = \{(1,2), (3,-1), (4,2), (-1,4)\} \quad g(x) = \{(-2,3), (1,-1), (3,2), (-1,0)\}$$

$$a) f(x) = \{(1,2), (\frac{3}{2}, -1), (2,2), (-\frac{1}{2}, 4)\}$$

$$b) f(x^2) = \{(1,2), (-1,2), (\sqrt{2}, -1), (-\sqrt{2}, -1), (2,2), (-2,2)\}$$

$$x^2=1 \rightarrow x=\pm 1 / x^2=2 \rightarrow x=\pm \sqrt{2} \\ x^2=4 \rightarrow x=\pm 2 \quad x^2=-1 \rightarrow \text{no}$$

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$$c) fg(x)+1 = \{(-2,19), (1,3), (2,9), (-1,1)\}$$

$$d) \frac{f}{g} = \{(1, \frac{4}{-1}), (2, \frac{-2}{2}), (-1, \frac{4}{0})\} \rightarrow \{(1, -4), (2, -1)\}$$

$$f(x) = \{(2,1), (3,1), (4,9), (-2,0)\} \quad g(x) = \{(5,1), (2,3), (-2,4), (-1,2), (3,3)\}$$

$$h = f+g \quad h = \{(2,5), (3,4), (-2,4)\}$$

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$$f(x) = \frac{\sqrt{4x}-x}{x-2} \quad g(x) = [x] + [-x] \quad \frac{f}{g+1}(x)$$

$$\sqrt{4x}-x \geq 0 \Rightarrow \sqrt{4x} \geq x \Rightarrow 4x \geq x^2 \rightarrow x^2-4x \leq 0 \rightarrow x(x-4) \leq 0 \quad \frac{4}{4-4+1} = 4 \quad [0, 4]$$

$$x-2 \neq 0 \Rightarrow x \neq 2 \quad D_f = [0, 2) \cup (2, 4]$$

$$\rightarrow g(x) = \begin{cases} 0 & x \in \mathbb{Z} \\ -1 & x \notin \mathbb{Z} \end{cases}$$

$$\frac{f(x)}{g(x)+1} \rightarrow g(x)+1 \neq 0 \rightarrow g(x) \neq -1 \Rightarrow D_g = \mathbb{Z} \Rightarrow D = \{0, 1, 2\}$$

5

$$f(x) = \sqrt{ax - x^2 - c}, \quad g(x) = \{(1, 1), (2, 2), (4, 4), (a, 1)\}$$

$$h(x) = \frac{f(x)}{g(x)} \quad D_f = \{(x^2 - ax + c) \geq 0\} \quad \frac{1}{-1} + \frac{1}{1-} \quad D_g = [1, 4]$$

$$h(x) = \left\{ (1, \frac{0}{1}), (2, \frac{\sqrt{2}}{2}), (4, 0) \right\} \quad f(1) = \sqrt{a-1-c} = 0 \quad f(2) = \sqrt{10-4-c} = \sqrt{6}$$

$$f(4) = \sqrt{16-16-c} = 0$$

$$h(x) = \left\{ (1, 0), (2, \frac{\sqrt{2}}{2}), (4, 0) \right\}$$

$$\text{الف) } y = x^2 - 2x^2 + 3x \rightarrow y = (x-1)^2 + 1 \Rightarrow y-1 = (x-1)^2 \rightarrow R_f = \mathbb{R}$$

$$\text{ب) } y = \frac{1}{x^2 - 2x} \rightarrow yx^2 - 2xy + 1 \rightarrow yx^2 - 2xy - 1 = 0 \rightarrow x = \frac{2y \pm \sqrt{(-2y)^2 - 4(-1)y}}{2y}$$

$$y \neq 0 \rightarrow 4y^2 + 4y \geq 0 \rightarrow 4y(y+1) \geq 0 \quad \frac{-1}{1} - \frac{0}{2} \quad D_f = (-\infty, -1] \cup (0, +\infty)$$

$$\text{الف) } y = x^2 - 4x + 1 \quad \min \left| \frac{1}{4} \right| \quad R_f = [1, +\infty)$$

$$\text{ب) } y = -x^2 + 4x + 3 \quad \max \left| \frac{1}{12} \right| \quad R_f = (-\infty, 12]$$

$$\text{ج) } y = \sqrt{x^2 - 4x - 4} \quad \min \left| \frac{1}{-4} \right| \quad R_f = \sqrt{(-\infty, -4]} \rightarrow [0, +\infty)$$

$$\text{د) } y = \sqrt{4x - x^2} \quad \max \left| \frac{1}{4} \right| \quad R_f = \sqrt{(-\infty, 4]} \rightarrow [0, 4]$$

$$\text{الف) } y = x^3 - 3x^2 + 2x + 1 \quad \text{نقطه‌های بحرانی توان فرد است} \rightarrow R_f = \mathbb{R}$$

$$\text{ب) } y = x^3 - 4x^2 + 3x + 2 \quad \text{نقطه‌های بحرانی توان فرد است} \rightarrow R_f = \mathbb{R}$$

$$\text{ج) } y = \sqrt{x^3 - 4x^2 + 3x + 1} \quad R_f = \sqrt{\mathbb{R}} \rightarrow [0, +\infty)$$

$$\text{د) } y = (x^3 - 4x^2 + 3x + 1)^2 \quad R_f = \mathbb{R}^+ \rightarrow [0, +\infty)$$

$$\text{الف) } y = \frac{3x+1}{x-2} \quad R_f = \mathbb{R} - \{3\}$$

$$\text{ب) } y = \frac{3x+1}{x+1} \quad R_f = \mathbb{R} - \{3\}$$