

$$f(x) = \sqrt{x-4} + 2 \Rightarrow (\sqrt{x-4} + 2)^2 = x - 4 + 4\sqrt{x-4} + 4 = x + 4\sqrt{x-4} = \sqrt{(x-4) + 4}^2$$

$$g(x) = \sqrt{x-4} - 2 \Rightarrow (\sqrt{x-4} - 2)^2 = x - 4 - 4\sqrt{x-4} + 4 = x - 4\sqrt{x-4} = \sqrt{(x-4) - 4}^2$$

$x \geq 4 \rightarrow \sqrt{x-4} \geq 2$

معادله درجه دوم را می توان به صورت $x^2 + bx + c = 0$ نوشت

$x \geq 4$

$$\sqrt{x-4} - 2 \geq 0$$

$f(x) = \sqrt{x-4} + 2$

$g(x) = \sqrt{x-4} - 2$

$f(x) + g(x) = (\sqrt{x-4} + 2) + (\sqrt{x-4} - 2) = 2\sqrt{x-4}$

$a=0, b=2, c=-4$

$\frac{0+2}{-4} = -\frac{1}{2}$

$x^2 - 5x + 4 = (x-1)(x-4) \rightarrow f \rightarrow x = 1, 4$

$D_f = \{0, 1, 2, 4\}$

$\rightarrow x=2 \quad g(2)=1 \quad f(2) = \frac{2+2}{2^2-5(2)+4} = \frac{4}{-2} = -2 \quad x=0 \quad g(0)=4$

$\frac{g(2)}{f(2)} = \frac{1}{-2} = -\frac{1}{2} \Rightarrow (2, -\frac{1}{2})$

$\frac{g(0)}{f(0)} = \frac{4}{4} = 1 \Rightarrow (0, 1)$

حالت نهایی: $\frac{g}{f} = \{(2, -\frac{1}{2}), (0, 1)\}$

$D_f = \{1, 4, -1\}$

$x=1 \rightarrow x = \frac{1}{f} \quad x=4 \rightarrow x = \frac{4}{f} \quad x=-1 \rightarrow x = -\frac{1}{f}$

$f(1)=2, f(4)=-1, f(-1)=4$

$f(x) = \{(1, 2), (4, -1), (-1, 4)\}$

$x^2 \in \{1, 4, -1\} \rightarrow x \in \{1, 2, -1\}$

$f(x^2) = \{(1, 2), (4, -1), (1, 2), (-1, 4)\}$

$x^2 = 1 \rightarrow x = \pm 1 \quad f(1)=2$

$x^2 = 4 \rightarrow x = \pm 2 \quad f(4)=-1$

$x^2 = -1 \rightarrow x = \pm i \quad f(-1)=4$

$h = rf + g$

$D_f = \{1, 4, -1\}$

$D_g = \{0, 1, 2, 4\}$

$A = \{1, 4, -1\}$

$x=1 \quad h(1) = 2f(1) + g(1) = 2(2) + 1 = 5$

$x=4 \quad h(4) = 2f(4) + g(4) = 2(-1) + 4 = 2$

$x=-1 \quad h(-1) = 2f(-1) + g(-1) = 2(4) + 0 = 8$

$h = \{(1, 5), (4, 2), (-1, 8)\}$

$f(x) = \frac{\sqrt{3x-x}}{x-2}, g(x) = [x] + [-x]$

$g(x) + 1 = \begin{cases} 1 & x \in \mathbb{Z} \\ 0 & x \notin \mathbb{Z} \end{cases}$

$g(x) + 1 \neq 0 \rightarrow x \in \mathbb{Z}$

$[x] = x \quad [-x] = -x \rightarrow x \in \mathbb{Z}$

$g(x) = 0$

$[x] + [-x] = -1 \rightarrow x \notin \mathbb{Z}$

$x-2 \neq 0 \rightarrow x \neq 2$

$D_{\frac{f}{g}} = \{0, 1, 2\}$

اعداد صحیح تا بی نهایت

$$Df = [1, 2]$$

$$5x - x^2 \geq 0$$

$$x(5-x) \geq 0 \Rightarrow 0 \leq x \leq 5$$

$$f(x) = \sqrt{-x^2 + 5x - 4}$$

$$x=1 \quad f(1) = \sqrt{5-1-4} = \sqrt{0} = 0 \quad \sqrt{4-4} = 0 \quad 1-4 = -3$$

$$h(1) = -\frac{1}{1} = -1 \Rightarrow x=2 \quad f(2) = \sqrt{10-4-4} = \sqrt{2} \quad \sqrt{4-4} = 0$$

$$h(2) = \frac{\sqrt{4-4}}{2} \Rightarrow x=4 \quad f(4) = \sqrt{20-16-4} = 0 \quad 2-4 = -2$$

$$h(4) = -\frac{2}{4} = -\frac{1}{2} \quad x=0 \quad f(0) = \sqrt{20-0-4} = 4 \quad -4 = -4$$

$$h(0) = -\frac{4}{0} = -\infty$$

$$f) y = x^2 - 4x + 4$$

$$x^2 - 4x + 4 = (x-2)^2 \Rightarrow y = (x-2)^2 \Rightarrow R_f = [0, \infty)$$

نکته: هر عدد حقیقی را می توان به صورت مربع یک عدد حقیقی نوشت.

$$b) x^2 - 4x = (x-2)^2 - 4 \quad y = \frac{1}{(x-2)^2 - 4} \quad t = (x-2)^2 - 4 \quad (x-2)^2 \geq 0$$

$$y = \frac{1}{t} \Rightarrow t \in [-4, 0) \cup (0, \infty) \Rightarrow y \in (-\infty, -\frac{1}{4}] \cup (\frac{1}{4}, \infty)$$

$$t \in (0, \infty) \Rightarrow y \in (0, \frac{1}{4}) \Rightarrow R_f = (-\infty, -\frac{1}{4}] \cup (0, \frac{1}{4}) \cup (\frac{1}{4}, \infty)$$

$$الف) x^2 - 4x + 10 \rightarrow y = (x-2)^2 + 6 \quad (x-2)^2 \geq 0 \quad y \geq 6 \quad R = [6, \infty)$$

$$ب) y = -(x^2 - 4x) + 4 \rightarrow y = -(x-2)^2 + 4 \rightarrow y = -(x-2)^2 + 4$$

$$(x-2)^2 \geq 0 \rightarrow -(x-2)^2 \leq 0 \quad y \leq 4 \quad R = (-\infty, 4]$$

$$ج) x^2 - 4x - 4 = (x-2)^2 - 8 \quad y \geq 0 \Rightarrow R = [8, \infty)$$

$$د) 4x - x^2 = 4 - (x-2)^2$$

$$y = \sqrt{4 - (x-2)^2} \quad 0 \leq 4 - (x-2)^2 \leq 4 \quad 0 \leq y \leq 2 \quad R = [0, 2]$$

$$الف) \rightarrow \text{مربع کامل} = 4 \quad R = [4, \infty)$$

$$ب) \rightarrow \text{مربع کامل} = 9 \quad R = [9, \infty)$$

$$ج) x^2 - 4x + 4x + 1 \rightarrow x^2 - 4x + 4x + 1 \rightarrow P(x) = x^2 - 4x + 4x + 1 \Rightarrow y = \sqrt{P(x)} \Rightarrow y \geq 0 \quad R = [0, \infty)$$

$$P(x) = x^2 - 4x + 4x + 1 \rightarrow R_p = R \quad y = [P(x)]^2 \Rightarrow y \geq 0 \quad R = [0, \infty)$$

$$y \neq \frac{a}{c} \quad الف) \frac{a}{c} = \frac{1}{1} = 1 \Rightarrow R = R - \{1\}$$

$$y = \frac{x+1}{x-2} \rightarrow y(x-2) = x+1 \quad yx - 2y = x+1$$

$$yx - 2y = x+1 \rightarrow x(y-2) = y+1 \Rightarrow x = \frac{y+1}{y-2} \Rightarrow y-2 \neq 0 \Rightarrow y \neq 2$$

$$ب) a=2, c=1 \quad \frac{a}{c} = \frac{2}{1} = 2 \quad \frac{a}{c} = 2 \Rightarrow R = R - \{2\}$$

$$y = \frac{x+1}{x+1} \rightarrow y(x+1) = x+1$$

$$yx + y = x + 1$$

$$yx - x = 1 - y \Rightarrow x = \frac{1-y}{y-1} \quad y-1 \neq 0 \quad y \neq 1 \quad R = R - \{1\}$$

$$2.) r g^r(x) + 1$$

مسألة

$$r [g(x)]^{r+1} \Rightarrow x = -2$$

$$g(-2) = 4 \quad r(4)^{r+1} = 18 + 1 = 19$$

$$x = 1 \quad g(1) = -1 \quad r(-1)^{r+1} = 4$$

$$x = 4 \quad g(4) = 2 \quad r(2)^{r+1} = 9$$

$$x = -1 \quad g(-1) = 0 \quad r(0)^{r+1} = 1$$

$$r g^r(x) + 1 = \{(-2, 19), (1, 4), (4, 9), (-1, 1)\}$$

$$D_f = \{1, 4, 9, -1\}$$

$$D_g = \{-2, 1, 4, -1\} \cap \{1, 4, -1\}$$

$$x = -1 \rightarrow g(-1) = 0 \rightarrow \text{تقريباً صفر} \Rightarrow x \neq -1$$

على قبل

$$x = 1 \quad \frac{r f(1)}{g(1)} = \frac{r(2)}{-1} = -4$$

$$x = 4 \quad \frac{r f(4)}{g(4)} = \frac{r(-1)}{2} = -1$$

$$\frac{r f}{g} = \{(1, -4), (4, -1)\}$$

سوال ۱ از راه محققه اکسترمم :

الف) $a > 0$

$$\begin{cases} x_{\min} = -\frac{b}{r_a} = -\frac{(-4)}{r(1)} = 1 \\ y_{\min} = r^2 - 4(r) + 1 = 1 - 4 + 1 = -2 \end{cases}$$

$\min y = -2 \rightarrow R = [-2, +\infty)$

ب) $a < 0$

$$\begin{cases} x_{\max} = -\frac{b}{r_a} = -\frac{-4}{r(-1)} = 1 \\ y_{\max} = -(1)^2 + 4(1) + 1 = -1 + 4 + 1 = 4 \end{cases} \rightarrow R = (-\infty, 4]$$

ج) $x^2 - 4x - 1$

$a > 0$

$$x_{\min} = -\frac{(-4)}{r(1)} = 1$$

$$r^2 - 4(r) - 1 = -1$$

$$x^2 - 4x - 1 \geq 0$$

$$y \geq 0$$

$$\Rightarrow R = [0, +\infty)$$

د) $4x - x^2 = -x^2 + 4x$ $a < 0$

$$x_{\max} = -\frac{-4}{r(-1)} = 1$$

$$4(1) - 1^2 = 3$$

$$-x^2 + 4x \leq 4$$

$$0 \leq y \leq 3$$

$$R = [0, 3]$$

$$f(x) = \left\{ \left(\frac{1}{2}, 2\right) \left(\frac{3}{2}, -1\right) (2, 2) \left(-\frac{1}{2}, 4\right) \right\} \quad \text{ن های } f \text{ باید حتماً برابر شوند!} \quad (3)$$

$$f(x) = \left\{ (1, 2) (-1, 2) (\sqrt{3}, -1) (-\sqrt{3}, -1) (2, 2) (-2, 2) \right\} \quad \text{همه ن های } f \text{ را برابر } x \text{ قرار دهیم!}$$

$$g(x) + 1 = \left\{ (-2, 19) (1, 3) (3, 9) (-1, 1) \right\} \quad \text{این عمل روی ی های و اعمال می شود:}$$

$$\frac{f}{g} = \left\{ \left(1, \frac{2 \times 2}{-1}\right) \left(3, \frac{2 \times -1}{2}\right) \left(-1, \frac{2 \times 4}{2}\right) \right\} = \left\{ (1, -2) (3, -1) \right\}$$

$$Df \rightarrow \Delta n - n^2 - 2 \geq 0 \rightarrow n^2 - \Delta n + 2 \leq 0 \rightarrow 1 \leq n \leq 4 \quad (4)$$

$$Df \cap Dg = \{1, 2, 3\}$$

$$\frac{f}{g} = h = \left\{ (1, 0) \left(2, \sqrt{\frac{2}{2}}\right) (3, 0) \right\}$$