

$$f(x) = \sqrt{x-4} + 2 \Rightarrow (\sqrt{x-4} + 2)^2 = x - 4 + 4\sqrt{x-4} + 4 = x + 4\sqrt{x-4} = \sqrt{(x-4) + 4}^2$$

$$g(x) = \sqrt{x-4} - 2 \Rightarrow (\sqrt{x-4} - 2)^2 = x - 4 - 4\sqrt{x-4} + 4 = x - 4\sqrt{x-4} = \sqrt{(x-4) - 4}^2$$

$x \geq 4 \rightarrow \sqrt{x-4} \geq 2$
 به عبارت داخل پرانتز زیر رادیکال مثبت است

$x \geq 4$
 $\sqrt{x-4} - 2 \geq 0$

$f(x) = \sqrt{x-4} + 2$
 $g(x) = \sqrt{x-4} - 2$

$f(x) + g(x) = (\sqrt{x-4} + 2) + (\sqrt{x-4} - 2) = 2\sqrt{x-4}$

$a=0, b=2, c=-4 \quad \frac{0+2}{-4} = -\frac{1}{2}$

$2^2 - 4x + 4 = (x-1)(x-4) \rightarrow f \rightarrow x = 1, 4$

$D_f = \{0, 1, 2, 4\}$

$\rightarrow x=2 \quad g(2)=1 \quad f(2) = \frac{2+2}{2^2-4(2)+4} = \frac{4}{4-8+4} = \frac{4}{0} = \infty$

$\frac{g(2)}{f(2)} = \frac{1}{\infty} = -\frac{1}{\infty} \rightarrow (2, -\frac{1}{\infty})$

$g(0) = 4$
 $f(0) = \frac{0+2}{0^2-4(0)+4} = \frac{2}{4} = \frac{1}{2}$
 $\frac{g(0)}{f(0)} = \frac{4}{\frac{1}{2}} = 4 \times \frac{2}{1} = 8 \rightarrow (0, 8)$

حالت نهایی: $\frac{g}{f} = \{(2, -\frac{1}{\infty}), (0, 8)\}$

$D_f = \{1, 4, -1, -\frac{1}{4}\}$

$2x=1 \rightarrow x=\frac{1}{2}, 2x=4 \rightarrow x=2, 2x=-1 \rightarrow x=-\frac{1}{2}, 2x=-\frac{1}{4} \rightarrow x=-\frac{1}{8}$

$f(1)=2, f(4)=-1, f(-1)=2, f(-\frac{1}{4})=4$

$f(x) = \{(\frac{1}{2}, 2), (2, -1), (-1, 2), (-\frac{1}{4}, 4)\}$

$x^2 \in \{1, 4, -1, -\frac{1}{4}\} \rightarrow x^2 \in \{1, 4\} \rightarrow x = \pm 1, \pm 2$

$f(x^2) = \{(-2, 2), (-1, 2), (1, 2), (2, 2)\}$

$h = 2f + g$

$D_f = \{1, 4, -1, -\frac{1}{4}\}$
 $D_g = \{0, 1, -2, -1, 4\}$
 $A = \{1, 4, -2\}$

$x=1 \quad h(1) = 2f(1) + g(1) = 2(2) + 1 = 5$

$x=4 \quad h(4) = 2f(4) + g(4) = 2(-1) + 4 = 2$

$x=-2 \quad h(-2) = 2f(-2) + g(-2) = 2(2) + 4 = 8$

$h = \{(1, 5), (4, 2), (-2, 8)\}$

$f(x) = \frac{\sqrt{3x-x}}{x-2}, g(x) = [x] + [-x]$

$g(x) + 1 = \begin{cases} 1 & x \in 2\mathbb{Z} \\ 0 & x \notin 2\mathbb{Z} \end{cases}$

$g(x) + 1 \neq 0 \rightarrow x \in 2\mathbb{Z}$

$[x] = x, [-x] = -x \rightarrow x \in 2\mathbb{Z}$
 $g(x) = 0$

$[x] + [-x] = -1 \rightarrow x \notin 2\mathbb{Z}$

$x-2 \neq 0 \rightarrow x \neq 2$

$D_{\frac{f}{g}} = \{0, 1, 4\}$

اعداد صحیح قابل تقبل: $0, 1, 2, 4$

$$\omega x - x^2 \geq 0$$

$$x(\omega - x) \geq 0 \Rightarrow 0 \leq x \leq \omega$$

$$x=1 \quad f(1) = \sqrt{\omega-1} - \epsilon = \sqrt{4} - \epsilon = 2 - \epsilon = -2$$

$$h(1) = -\frac{r}{\mu} = -\frac{r}{\mu} \Rightarrow x=r \quad f(r) = \sqrt{10-\epsilon} - \epsilon = \sqrt{4-\epsilon}$$

$$h(r) = \frac{\sqrt{4-\epsilon}}{r} \Rightarrow x=\epsilon \quad f(\epsilon) = \sqrt{r_0-14} - \epsilon = r-\epsilon = -2$$

$$h(\epsilon) = -\frac{r}{4} = -\frac{1}{4} \quad x=0 \quad f(0) = \sqrt{r_0-r_0} - \epsilon = -\epsilon$$

$$h(0) = -\frac{\epsilon}{1} = -\epsilon$$

$$h = \left\{ (1, -\frac{r}{\mu}), (r, \frac{\sqrt{4-\epsilon}}{r}), (\epsilon, -\frac{1}{4}), (0, -\epsilon) \right\}$$

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$$f) y = x^2 - 4x + 4$$

$$x^2 - 4x + 4 = (x-2)^2 \Rightarrow y = (x-2)^2 \Rightarrow R_f = [0, +\infty)$$

$$x^2 - 4x = (x-2)^2 - 4 \quad y = \frac{1}{(x-2)^2 - 4} \quad t = (x-2)^2 - 4 \quad (x-2)^2 \geq 0$$

$$y = \frac{1}{t} \Rightarrow t \in [-1, 0) \Rightarrow y \in (-\infty, -1] \quad t \in [0, +\infty) \Rightarrow y \in (0, +\infty)$$

$$t \in (0, +\infty) \Rightarrow y \in (0, +\infty) \Rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$$

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$$الف) x^2 - 4x + 10 \rightarrow y = (x-2)^2 + 6 \quad (x-2)^2 \geq 0 \quad y \geq 6 \quad R = [6, +\infty)$$

$$ب) y = -(x^2 - 4x) + 4 \rightarrow y = -(x-2)^2 + 4 \rightarrow y = -(x-2)^2 + 4$$

$$(x-2)^2 \geq 0 \rightarrow -(x-2)^2 \leq 0 \quad y \leq 4 \quad R = (-\infty, 4]$$

$$ج) x^2 - 4x - 4 = (x-2)^2 - 8 \quad y \geq 0 \Rightarrow R = [0, +\infty)$$

$$د) 4x - x^2 = 4 - (x-2)^2$$

$$y = \sqrt{4 - (x-2)^2} \quad 0 \leq 4 - (x-2)^2 \leq 4 \quad 0 \leq y \leq 2 \quad R = [0, 2]$$

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$$الف) \rightarrow \text{مربع کامل} = 4 \quad R = \{4\}$$

$$ب) \rightarrow \text{مربع کامل} = 9 \quad R = \{9\}$$

$$ج) x^2 - 4x + 4x + 1 \rightarrow \text{مربع کامل} \rightarrow P(x) = 1 \Rightarrow y = \sqrt{P(x)} \Rightarrow y \geq 0 \quad R = [0, +\infty)$$

$$P(x) = x^2 - 4x + 4x + 1 \rightarrow \text{مربع کامل} \quad R_P = R \quad y = [P(x)]^2 \rightarrow y \geq 0 \Rightarrow R = [0, +\infty)$$

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$$y \neq \frac{a}{c} \quad الف) \frac{a}{c} = \frac{r}{1} = r \Rightarrow R = R - \{r\}$$

$$y = \frac{rx+1}{x-r} \rightarrow y(x-r) = rx+1$$

$$yx - ry = rx+1$$

$$yx - rx = ry+1 \rightarrow x(y-r) = ry+1 \Rightarrow x = \frac{ry+1}{y-r} \Rightarrow y-r \neq 0$$

$$ب) a=r, c=1 \quad \frac{a}{c} = \frac{r}{1} = r \quad \frac{a}{c} = r \Rightarrow R = R - \{r\}$$

$$y = \frac{rx+1}{x+1} \rightarrow y(x+1) = rx+1$$

$$yx+y = rx+1$$

$$yx - rx = 1-y \Rightarrow x = \frac{1-y}{y-r}$$

$$x(y-r) = 1-y \quad y-r \neq 0 \quad y \neq r \quad R = R - \{r\}$$

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$$2.) r g^r(x) + 1$$

مسألة

$$r [g(x)]^{r+1} \Rightarrow x = -2$$

$$g(-2) = 4 \quad r(4)^{r+1} = 18 + 1 = 19$$

$$x = 1 \quad g(1) = -1 \quad r(-1)^{r+1} = 4$$

$$x = 4 \quad g(4) = 2 \quad r(2)^{r+1} = 9$$

$$x = -1 \quad g(-1) = 0 \quad r(0)^{r+1} = 1$$

$$r g^r(x) + 1 = \{(-2, 19), (1, 4), (4, 9), (-1, 1)\}$$

$$1) D_f = \{1, 4, 9, -1\}$$

$$D_g = \{-2, 1, 4, -1\} \cap \{1, 4, -1\}$$

$$x = -1 \rightarrow g(-1) = 0 \rightarrow \text{تقريباً مدونه} \Rightarrow x \neq -1$$

على قبل

$$x = 1 \quad \frac{r f(1)}{g(1)} = \frac{r(1)}{-1} = -4$$

$$x = 4 \quad \frac{r f(4)}{g(4)} = \frac{r(-1)}{2} = -1$$

$$\frac{r f}{g} = \{(1, -4), (4, -1)\}$$

سوال ۱ از راه محققه اکتصاف :

الف) $a > 0$

$$\begin{cases} x_{\min} = -\frac{b}{r_a} = -\frac{(-4)}{r(1)} = 1 \\ y_{\min} = r^2 - 4(r) + 1 = 4 - 4 + 1 = 1 \end{cases}$$

$\min y = 1 \rightarrow R = [1, +\infty)$

ب) $a < 0$

$$\begin{cases} x_{\max} = -\frac{b}{r_a} = -\frac{-4}{r(-1)} = 1 \\ y_{\max} = -(1)^2 + 4(1) + 1 = -1 + 4 + 1 = 4 \end{cases} \rightarrow R = (-\infty, 4]$$

ج) $x^2 - 4x - 1$

$a > 0$

$$x_{\min} = -\frac{(-4)}{r(1)} = 1$$

$$r^2 - 4(r) - 1 = -1$$

$$x^2 - 4x - 1 \geq 0$$

$$y \geq 0$$

$$\Rightarrow R = [0, +\infty)$$

د) $4x - x^2 = -x^2 + 4x$ $a < 0$

$$x_{\max} = -\frac{-4}{r(-1)} = 1$$

$$4(1) - 1^2 = 3$$

$$-x^2 + 4x \leq 4$$

$$0 \leq y \leq 3$$

$$R = [0, 3]$$