

$$f(x) = \sqrt{x+4}\sqrt{x-4} \quad \text{و} \quad g(x) = \sqrt{x-4} + 2$$

$$f(x) + g(x) = \sqrt{x-4} + 2 + \sqrt{x-4} - 2 = 2\sqrt{x-4} = a + b\sqrt{x+c}$$

$$a=0 \quad b=2 \quad c=-4 \Rightarrow \frac{a+b}{c} = \frac{2}{-4} = -\frac{1}{2}$$

$$f(x) = \frac{x+2}{x^2-4x+3} \Rightarrow D_f = \{x \mid x^2-4x+3 \neq 0\} = \{x \mid (x-1)(x-3) \neq 0\} = \mathbb{R} - \{1, 3\}$$

$$g(x) = \{(2, 1), (1, 0), (3, 0), (0, 4)\} \Rightarrow D_g = \{2, 1, 3, 0\}$$

$$D_{f \circ g} = D_g \cap D_f = \{x \mid f(g(x)) \text{ معنی دار است}\} = \{2, 0\}$$

$$\frac{f}{g} = \left\{ \left(2, \frac{1}{-1}\right), \left(0, \frac{4}{3}\right) \right\} = \left\{ \left(2, -1\right), \left(0, \frac{4}{3}\right) \right\}$$

$$a) f(x) = \left\{ \left(\frac{1}{2}, 2\right), \left(\frac{3}{2}, -1\right), (2, 2), \left(-\frac{1}{2}, 2\right) \right\}$$

$$b) f(x^2) = \{(1, 2), (-1, 2), (\sqrt{2}, -1), (-\sqrt{2}, -1), (2, 2), (-2, 2)\}$$

$$c) 2g(x) + 1 = \{(-2, 9), (1, 3), (3, 9), (-1, 9)\}$$

$$d) \frac{f}{g} = \{(1, -4), (3, -1), (-1, -1)\} = \{(1, -4), (3, -1)\}$$

$$h = 2f + g$$

$$2f + g = \{(2, 5), (-2, 4), (3, 5)\}$$

$$g = \{(5, 1), (2, 3), (-2, 2), (-1, 2), (4, 2)\}$$

$$f(x) = \{(2, 1), (3, 1), (4, 9), (-2, 0)\}$$

$$f(x) = \frac{\sqrt{3x} - x}{x-2}, \quad g(x) = [x] + [-x]$$

$$\Rightarrow \left(\frac{f}{g+1}\right)(x) = \frac{\sqrt{3x} - x}{x-2} \div ([x] + [-x] + 1)$$

$$\Rightarrow \begin{cases} 1) 3x \geq 0 \Rightarrow x \geq 0 \\ 2) \sqrt{3x} - x \geq 0 \Rightarrow \sqrt{3x} \geq x \Rightarrow x^2 - 3x \leq 0 \Rightarrow [0, 3] \\ 3) x-2 \neq 0 \Rightarrow x \neq 2 \\ 4) [x] + [-x] + 1 \neq 0 \Rightarrow \begin{cases} [x] + [-x] = -1 & \text{if } x \notin \mathbb{Z} \\ [x] + [-x] = 0 & \text{if } x \in \mathbb{Z} \end{cases} \end{cases}$$

$$\Rightarrow [x] + [-x] + 1 \neq 0 \Rightarrow x \in \mathbb{Z}$$

$$\Rightarrow D_{\frac{f}{g+1}} = \{0, 1, 3\}$$

$f(x) = \sqrt{-x^2 + 5x - 4} \quad g(x) = \{(1,3), (2,2), (4,4), (5,1)\}$ $h(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{-x^2 + 5x - 4}}{g(x)}$ ① $-x^2 + 5x - 4 \geq 0 \Rightarrow x^2 - 5x + 4 \leq 0 \Rightarrow (x-1)(x-4) \leq 0 \Rightarrow \begin{matrix} 1 & & 4 \\ & & \\ + & & + \end{matrix} \Rightarrow [1, 4]$ ② $g(x) \neq 0 \Rightarrow x \neq 0 \Rightarrow D_h = \{1, 2, 4\} \Rightarrow h = \{(1,0), (2, \frac{2}{\sqrt{2}}), (4,0)\}$	6
الف) $y = x^3 - 3x^2 + 3x - 1 + 1 = (x-1)^3 + 1$ $\Rightarrow y-1 = (x-1)^3 \Rightarrow x-1 = \sqrt[3]{y-1} \Rightarrow x = \sqrt[3]{y-1} + 1 \Rightarrow R_f = \mathbb{R}$ ب) $y = \frac{1}{x^2 - x} \Rightarrow yx^2 - yx - 1 = 0$ $\begin{cases} y=0 \Rightarrow -1=0 \text{ غلط} \\ y \neq 0 \Rightarrow x = \frac{y \pm \sqrt{y^2 + 4y}}{2y} \Rightarrow y^2 + 4y \geq 0 \Rightarrow y(y+4) \geq 0 \end{cases}$ $\Rightarrow R_f = \mathbb{R} - (-1, 0] = (-\infty, -1] \cup (0, +\infty)$ نکته: آزمون تناسلی که اینجوری کامل نمیشه بدونه	7
الف) $\min \left \frac{y}{4} \right \Rightarrow R_f = [4, +\infty)$ $\rightarrow$ سیمین ترین مقدار آن 4 است پس بیشتر از آن هست ب) $\max \left \frac{y}{12} \right \Rightarrow R_f = (-\infty, 12]$ $\rightarrow$ بزرگترین مقدار آن 12 است پس بیشتر از آن کمتر از 12 است ج) $\min \left \frac{y}{-7} \right \Rightarrow [-7, +\infty) \xrightarrow{\text{زیرارائه}} R_f = [0, +\infty)$ د) $\min \left \frac{y}{9} \right \Rightarrow (-\infty, 9] \xrightarrow{\text{زیرارائه}} R_f = [0, 9]$	8
الف) $y = x^3 - 5x^2 + 2x + 1 \rightarrow$ بیضه‌ای شکل $\Rightarrow R_f = \mathbb{R}$ ب) $y = x^3 - 4x^2 + 3x + 2 \rightarrow \dots \Rightarrow R_f = \mathbb{R}$ ج) $y = \sqrt{x^3 - 4x^2 + 5x + 1} \rightarrow$ بیضه‌ای شکل عبارت زیر رادیکال $\rightarrow \mathbb{R} \xrightarrow{\text{زیرارائه}} R_f = [0, +\infty)$ د) $y = (x^3 - 4x^2 + 5x + 1)^2 \rightarrow$ بیضه‌ای شکل عبارت داخل $\rightarrow \mathbb{R} \xrightarrow{\text{زیرارائه}} R_f = [0, +\infty)$	9
الف) $y = \frac{3x+1}{x-2}$ $\begin{cases} \text{رابطه معکوس: } x = -\frac{3y-1}{y-3} = \frac{3y+1}{y-3} \Rightarrow y \neq 3 \Rightarrow R_f = \mathbb{R} - \{3\} \\ \text{از آنجا که } y \text{ محدود نیست: } R_f = \mathbb{R} - \{3\} \Rightarrow R_f = \mathbb{R} - \{3\} \end{cases}$ ب) $y = \frac{2x+1}{x+1}$ $\begin{cases} \text{رابطه معکوس: } x = -\frac{y-1}{y-2} = \frac{1-y}{y-2} \Rightarrow y \neq 2 \Rightarrow R_f = \mathbb{R} - \{2\} \\ \text{رابطه معکوس: } R_f = \mathbb{R} - \{2\} \end{cases}$	10