

$$f(x) = \sqrt{x+4\sqrt{x-1}} \xrightarrow{\text{نویز اول}} f(x) = \sqrt{x-1+4\sqrt{x-1}+4} = \sqrt{(x-1+2)^2} = |\sqrt{x-1}+2| = \sqrt{x-1}+2$$

$$g(x) = \sqrt{x-4\sqrt{x-1}} \xrightarrow{\text{نویز دوم}} g(x) = \sqrt{x-1-4\sqrt{x-1}+4} = \sqrt{(x-1-2)^2} = |\sqrt{x-1}-2| \xrightarrow{\text{در این صورت}} \sqrt{x-1}-2$$

$$f(x) + g(x) = \sqrt{x-1}+2 + \sqrt{x-1}-2 = 2\sqrt{x-1} = a + b\sqrt{x+c}$$

$$a=0 \quad b=2 \quad c=-1 \Rightarrow \frac{a+b}{c} = \frac{2}{-1} = -2$$

$$f(x) = \frac{x+2}{x^2-4x+3} \Rightarrow \text{دست آید} \Rightarrow x^2-4x+3 \neq 0 \quad (x-1)(x-3) \neq 0 \Rightarrow D_f = \mathbb{R} - \{1, 3\}$$

$$g(x) = \{(2, 1), (1, 0), (3, 0), (0, 4)\} \Rightarrow D_g = \{2, 1, 3, 0\}$$

$$D_{\frac{f}{g}} = D_g \cap D_f - \{x \mid f(x)=0\} = \{2, 0\}$$

$$\frac{f}{g} = \left\{ \left(2, \frac{1}{-4}\right), \left(0, \frac{4}{3}\right) \right\} = \left\{ \left(2, -\frac{1}{4}\right), \left(0, \frac{4}{3}\right) \right\}$$

$$\text{الف)} f(x) = \left\{ \left(\frac{1}{2}, 2\right), \left(\frac{3}{2}, -1\right), (2, 2), \left(-\frac{1}{2}, 2\right) \right\}$$

$$\text{ب)} f(x^2) = \{(1, 2), (-1, 2), (\sqrt{2}, -1), (-\sqrt{2}, -1), (2, 2), (-2, 2)\}$$

$$\text{ج)} 2g(x)+1 = \{(-2, 1), (1, 3), (3, 1), (-1, 1)\}$$

$$\text{د)} \frac{f}{g} = \{(1, -4), (3, -1), \left(-\frac{1}{2}, 2\right)\} = \{(1, -4), (3, -1)\}$$

$$h = 2f + g$$

$$2f + g = \{(2, 5), (-2, 4), (3, 5)\}$$

$$g = \{(5, 1), (2, 3), (-2, 1), (-1, 2), (3, 2)\}$$

$$f(x) = \{(2, 1), (3, 1), (4, 9), (-2, 0)\}$$

$$f(x) = \frac{\sqrt{3x}-x}{x-2}, \quad g(x) = [x] + [-x]$$

$$\Rightarrow \left(\frac{f}{g+1}\right)(x) = \frac{\sqrt{3x}-x}{x-2} \div [x] + [-x] + 1$$

$$\Rightarrow \text{① } 3x \geq 0 \Rightarrow x \geq 0$$

$$\text{② } \sqrt{3x}-x \geq 0 \Rightarrow \sqrt{3x} \geq x \Rightarrow x^2-3x \leq 0 \Rightarrow [0, 3]$$

$$\text{③ } x-2 \neq 0 \Rightarrow x \neq 2$$

$$\text{④ } [x] + [-x] + 1 \neq 0 \Rightarrow \begin{cases} [x] + [-x] = -1 & \text{if } x \notin \mathbb{Z} \\ [x] + [-x] = 0 & \text{if } x \in \mathbb{Z} \end{cases}$$

$$\Rightarrow [x] + [-x] \neq -1 \Rightarrow x \in \mathbb{Z}$$

$$\Rightarrow D_{\frac{f}{g+1}} = \{0, 1, 3\}$$

$f(x) = \sqrt{-x^2 + 8x - 8}$ $g(x) = \{(1, 3), (2, 2), (4, 4), (5, 1)\}$ $h(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{-x^2 + 8x - 8}}{g(x)}$ الف) $-x^2 + 8x - 8 \geq 0 \Rightarrow x^2 - 8x + 8 \leq 0 \Rightarrow (x-1)(x-7) \leq 0 \Rightarrow \begin{matrix} 1 & 7 \\ & \\ + & - & + \end{matrix} \Rightarrow [1, 7]$ ب) $g(x) \neq 0 \Rightarrow x \neq 0 \Rightarrow D_h = \{1, 2, 4\} \Rightarrow h = \{(1, 0), (2, \frac{2}{\sqrt{2}}), (4, 0)\}$	6
الف) $y = x^3 - 3x^2 + 3x - 1 \stackrel{+1}{\Rightarrow} y = x^3 - 3x^2 + 3x - 1 + 1 = (x-1)^3 + 1$ $\Rightarrow y-1 = (x-1)^3 \Rightarrow x-1 = \sqrt[3]{y-1} \Rightarrow x = \sqrt[3]{y-1} + 1 \Rightarrow R_f = \mathbb{R}$ ب) $y = \frac{1}{x^2 - x} \Rightarrow yx^2 - yx - 1 = 0$ $\begin{cases} y=0 \Rightarrow -1=0 \text{ غلط} \\ y \neq 0 \Rightarrow x = \frac{y \pm \sqrt{y^2 + 4y}}{2y} \end{cases} \Rightarrow y^2 + 4y \geq 0 \Rightarrow y(y+4) \geq 0$ $\begin{matrix} & 0 & -4 \\ & & \\ + & - & + \end{matrix}$ $\Rightarrow R_f = \mathbb{R} - (-1, 0] = (-\infty, -1] \cup (0, +\infty)$	7
الف) $\min \left \frac{y}{4} \right \Rightarrow R_f = [4, +\infty) \rightarrow$ <i>محدودترین مقدار آن 4 است پس بیشتر از آن است</i> ب) $\max \left \frac{y}{12} \right \Rightarrow R_f = (-\infty, 12] \rightarrow$ <i>محدودترین مقدار آن 12 است پس بیشتر از آن کمتر از 12 است</i> ج) $\min \left \frac{y}{-7} \right \Rightarrow [-7, +\infty) \xrightarrow{\text{زیرارائه}} R_f = [0, +\infty)$ د) $\min \left \frac{y}{9} \right \Rightarrow (-\infty, 9] \xrightarrow{\text{زیرارائه}} R_f = [0, 9]$	8
الف) $y = x^3 - 5x^2 + 2x + 1 \rightarrow$ <i>بزرگترین مقدار آن 1 است</i> $\Rightarrow R_f = \mathbb{R}$ ب) $y = x^3 - 4x^2 + 3x + 2 \rightarrow \dots \Rightarrow R_f = \mathbb{R}$ ج) $y = \sqrt{x^3 - 4x^2 + 5x + 1} \rightarrow$ <i>بزرگترین مقدار آن عبارت از 1 است</i> $\xrightarrow{\text{زیرارائه}} R_f = [0, +\infty)$ د) $y = (x^3 - 4x^2 + 5x + 1)^2 \rightarrow$ <i>بزرگترین مقدار آن عبارت از 1 است</i> $\xrightarrow{\text{زیرارائه}} R_f = [0, +\infty)$ $\because y \geq 0$ <i>له وجهی داریم</i>	9
الف) $y = \frac{3x+1}{x-2}$ $\begin{cases} \text{رابطه معادله: } x = -\frac{3y-1}{y-2} = \frac{3y+1}{y-2} \Rightarrow y \neq 2 \Rightarrow R_f = \mathbb{R} - \{2\} \\ \text{از آنجا که } y \text{ محدود نیست: } R_f = \mathbb{R} - \{2\} \Rightarrow R_f = \mathbb{R} - \{2\} \end{cases}$ ب) $y = \frac{2x+1}{x+1}$ $\begin{cases} \text{رابطه معادله: } x = -\frac{y-1}{y-2} = \frac{1-y}{y-2} \Rightarrow y \neq 2 \Rightarrow R_f = \mathbb{R} - \{2\} \\ \text{رابطه معادله: } R_f = \mathbb{R} - \{2\} \end{cases}$	10