

روز هفته : نام و نام خانوادگی نوشته شده است ! نمره شما صفر باشد ! لطفاً مرتباً : تاریخ : / /

①

$$f(x) = \sqrt{x+4}\sqrt{x-4} \quad x > 4$$

$$g(x) = \sqrt{x-4}\sqrt{x-4} \quad f(x) + g(x) = a + b\sqrt{x+c}$$

$$t = \sqrt{x-4}$$

$$x > 4 \Rightarrow t > \sqrt{4-4} = 0 \Rightarrow x = t^2 + 4$$

$$f(x) = \sqrt{x+4}\sqrt{x-4} = \sqrt{(t^2+4)+4}t = \sqrt{(t+2)^2} = t+2$$

$$g(x) = \sqrt{x-4}\sqrt{x-4} = \sqrt{(t^2+4)-4}t = \sqrt{t^2} = t$$

$$= \sqrt{(t-2)^2} = t-2$$

$$f(x) + g(x) = (t+2) + (t-2) = 2t = 2\sqrt{x-4}$$

$$2\sqrt{x-4} = 0 + 2\sqrt{x+(-4)} \Rightarrow a=0, b=2, c=-4$$

$$\frac{a+b}{c} = \frac{0+2}{-4} = -\frac{1}{2} \quad \text{②}$$

②

$$f(x) = \frac{x+2}{x^2-4x+3}, \quad g(x) = \{(2,1), (1,0), (3,0),$$

$$(0,2)\} \quad \frac{g}{f} = ?$$

$$D_g = \{0, 1, 2, 3\}$$

$$x^2 - 4x + 3 \neq 0 \Rightarrow (x-1)(x-3) \neq 0 \Rightarrow x \neq 1, 3$$

$$D_f = R - \{1, 3\}$$

$$D_{f,g} = D_g \cap D_f = \{1, 2, 3, 0\} \cap (R - \{1, 3\}) = \{0, 2\}$$

$$x=0 \Rightarrow f(0) = \frac{0+2}{0-0+3} = \frac{2}{3}, g(0) = 4$$

$$\frac{g(0)}{f(0)} = \frac{4}{\frac{2}{3}} = 4 \times \frac{3}{2} = 6 \Rightarrow (0, 6)$$

$$x=2 \Rightarrow f(2) = \frac{2+2}{4-1+3} = -1, g(2) = 1$$

$$\frac{g(2)}{f(2)} = -1 \Rightarrow (2, -1) \quad \left\{ (0, 6), (2, -1) \right\}$$

$$\textcircled{3} f(x) = \{(1, 2), (2, -1), (3, 2), (-1, 4)\}$$

$$g(x) = \{(-2, 3), (1, -1), (3, 2), (-1, 0)\}$$

$$\text{الف) } f(1/x) = \left\{ \left(\frac{1}{1}, 2\right), \left(\frac{1}{2}, -1\right), \left(\frac{1}{3}, 2\right), \left(-\frac{1}{1}, 4\right) \right\}$$

$$1/x=1 \Rightarrow x=1, f(1)=2$$

$$1/x=2 \Rightarrow x=\frac{1}{2}, f\left(\frac{1}{2}\right)=-1$$

$$2a = 2 \Rightarrow a = 1, f(1) = 2$$

$$2a = -1 \Rightarrow a = -\frac{1}{2}, f(-1) = 9$$

$$\rightarrow f(x) = \{(1, 2), (-1, 2), (2, 2), (-2, 2), (\sqrt{2}, 1), (-\sqrt{2}, 1)\}$$

$$a^2 = 1 \Rightarrow a = \pm 1, f(1) = 2$$

$$a^2 = 2 \Rightarrow a = \pm \sqrt{2}, f(\sqrt{2}) = 1$$

$$a^2 = 3 \Rightarrow a = \pm \sqrt{3}, f(\sqrt{3}) = -1$$

$$2) g^2(x) + 1 = \{(-2, 19), (-1, 3), (1, 9), (0, 1)\}$$

$$x = -2 \Rightarrow g(-2) = 3 \Rightarrow 2x^2 + 1 = 19$$

$$x = -1 \Rightarrow g(-1) = -1 \Rightarrow 2x^2 + 1 = 3$$

$$x = 1 \Rightarrow g(1) = 2 \Rightarrow 2x^2 + 1 = 9$$

$$x = 0 \Rightarrow g(0) = 0 \Rightarrow 2x^2 + 1 = 1$$

$$3) \frac{f}{g} = \{(1, -1), (2, 1)\}$$

$$Df \cap Dg = \{1, -1, 3\} \quad g(x) \neq 0$$

$$D_{\frac{f}{g}} = \{1, 3\}$$

$$x = 1 \Rightarrow \frac{f(1)}{g(1)} = \frac{2}{-1} = -2$$

$$x = 3 \Rightarrow \frac{f(3)}{g(3)} = \frac{-1}{2} = -\frac{1}{2}$$

$$\textcircled{1} f(x) = \{(1, 1), (2, 1), (3, 9), (-1, 0)\}$$

$$g(x) = \{(0, 1), (2, 3), (-2, 9), (-1, 2), (3, 3)\}$$

$$h = f + g \quad \text{DF} = \{1, 2, 3, -1\}$$

$$\text{D}g = \{0, 2, -2, -1, 3\}$$

$$\text{DF} = \text{D}g \cap \text{DF} = \{1, 2, -1\}$$

$$x=1 \Rightarrow h(1) = f(1) + g(1) = 1(1) + 3 = 1+3=4$$

$$x=2 \Rightarrow h(2) = f(2) + g(2) = 1(1) + 3 = 1+3=4$$

$$x=-1 \Rightarrow h(-1) = f(-1) + g(-1) = 1(0) + 4 = 0+4=4$$

$$h = \{(1, 4), (2, 4), (-1, 4)\} \quad \textcircled{2}$$

$$\textcircled{2} \sqrt{\frac{ra-a}{a-r}} = f(x), g(x) = [x] + [-x]$$

$$\left(\frac{f}{g+1}\right)(x)$$

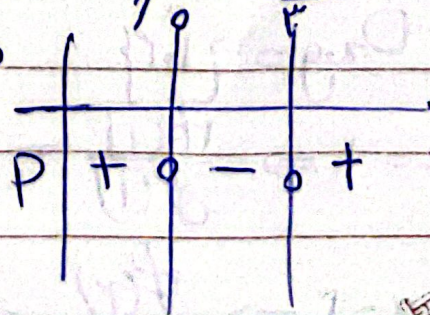
ممنون

$$\sqrt{ra-a} \geq 0 \Rightarrow \sqrt{ra} \geq a \Rightarrow ra \geq a^2 \Rightarrow$$

$$a^2 - ra \leq 0 \Rightarrow a(a-r) \leq 0$$

$$[0, r]$$

$$a-r \neq 0 \Rightarrow a \neq r$$



$$D_f = [0, 4] - \{2\}$$

$$g(x) = [x] + [-x]$$

$$\Rightarrow \text{برای } x \text{ صحیح} \quad [x] + [-x] = x - x = 0$$

$$\Rightarrow \text{برای } x \text{ صحیح نباشد} \quad [x] + [-x] = -1$$

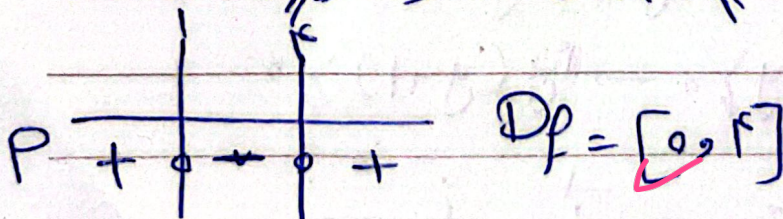
$$g(x) + 1 \neq 0 \Rightarrow \text{برای } x \text{ صحیح نباشد}$$

$$x \in \{0, 1, 3\}$$

$$\textcircled{1} f(x) = \sqrt{4x - x^2 - 4}, \quad g(x) = \{(1, 2)(2, 2)(3, 4)(4, 1)\}$$

$$h(x) = \frac{f(x)}{g(x)} = ?$$

$$4x - x^2 - 4 \geq 0 \Rightarrow x^2 - 4x + 4 \leq 0 \Rightarrow (x-2)(x-2) \leq 0$$



$$D_g = \{1, 2, 3, 4\} \quad D_h = D_f \cap D_g = \{2\}$$

$$\Rightarrow a=1 \quad f(1)=\sqrt{a(1)-1^2-1}=0 \quad (1,0)$$

$$\Rightarrow a=2 \quad f(2)=\sqrt{a(2)-2^2-1}=\sqrt{1} \quad (2, \frac{\sqrt{1}}{2})$$

$$\Rightarrow a=3 \quad f(3)=\sqrt{a(3)-3^2-1}=\sqrt{0}=0 \quad (3,0)$$

$$g(1)=3$$

$$h(a)=\{(1,0), (2, \frac{\sqrt{1}}{2}), (3,0)\}$$

$$\textcircled{1} \text{ الف) } y=(a^3-3a^2+3a-1)+1$$

$$y=(a-1)^3+1 \Rightarrow (a-1)^3=y-1 \Rightarrow \sqrt[3]{y-1}=a-1$$

$$\Rightarrow a=\sqrt[3]{y-1}+1 \quad R_f=\mathbb{R}$$

$$y=\frac{1}{a^2-2a} \Rightarrow y \neq 0$$

$$y(a^2-2a)=1 \Rightarrow ya^2-2ya-1=0$$

$$\Delta \geq 0 \Rightarrow 4y^2+4y \geq 0 \Rightarrow y(y+1) \geq 0$$

$$\Rightarrow y(y+1) \geq 0$$

$$\begin{array}{c} -1 \quad 0 \\ | \quad | \\ + \quad - \quad + \end{array}$$

$$R_g=(-\infty, -1] \cup (0, +\infty)$$

سوال ۸ و ۹ ؟

$$\textcircled{10} \text{ الف) } y = \frac{3x+1}{x-2}$$

$$\text{مخرج} \Rightarrow \frac{ax+b}{cx+d} = y \quad \frac{a}{c} = \frac{r}{1} = r$$

$$R = \mathbb{R} \setminus \{2\}$$

$$\text{ب) } y = \frac{2x+1}{x+1}$$

$$\text{مخرج} \Rightarrow \frac{ax+b}{cx+d} = y \quad \frac{a}{c} = \frac{r}{1} = r$$

$$R = \mathbb{R} \setminus \{1\} \quad \textcircled{11}$$

(۹) توابع درجه فرد جبری برابر ۱ دارند!

الف) $Rf \subseteq \mathbb{R}$

ب) $Rf \subseteq \mathbb{R}$

ج) $Rf \subseteq \sqrt{\mathbb{R}} = [0, +\infty)$

د) $Rf \subseteq (\mathbb{R})^2 = [0, +\infty)$

الف-۱) $y = x^2 - 2x + 1 \rightarrow \min \begin{cases} x = \frac{2}{2} = 1 \\ y = 1 - 2 + 1 = 0 \end{cases} \rightarrow Rf = [0, +\infty)$

ب-۱) $y = -x^2 + 4x + 3 \rightarrow \max \begin{cases} x = \frac{-4}{-2} = 2 \\ y = -9 + 18 + 3 = 12 \end{cases} \rightarrow Rf = (-\infty, 12]$

ج-۱) $y = x^2 - 2x - 3 \rightarrow \min \begin{cases} x = \frac{2}{2} = 1 \\ y = 1 - 2 - 3 = -4 \end{cases} \rightarrow Rf = \sqrt{[-4, +\infty)} = [0, +\infty)$

د-۱) $y = 4x - x^2 \rightarrow \max \begin{cases} x = \frac{-4}{-2} = 2 \\ y = 18 - 9 = 9 \end{cases} \rightarrow Rf = \sqrt{(-\infty, 9]} = [0, 9]$