

①

$$f(x) = \sqrt{x+r}\sqrt{x-r} \quad x > r$$

$$g(x) = \sqrt{x-r}\sqrt{x-r} \quad f(x) + g(x) = a + b\sqrt{x+c}$$

$$t = \sqrt{x-r}$$

$$x > r \Rightarrow t > \sqrt{1-r} = r \Rightarrow x = t^2 + r$$

$$f(x) = \sqrt{x+r}\sqrt{x-r} = \sqrt{(t^2+r)+r}t = \sqrt{(t+r)r} = t+r$$

$$g(x) = \sqrt{x-r}\sqrt{x-r} = \sqrt{(t^2+r)-r}t = \sqrt{t^2-r}t = t-r$$

$$= \sqrt{(t-r)r} = t-r$$

$$f(x) + g(x) = (t+r) + (t-r) = rt = r\sqrt{x-r}$$

$$r\sqrt{x-r} = 0 + r\sqrt{x+(-r)} \Rightarrow a=0, b=r, c=-r$$

$$\frac{a+b}{c} = \frac{0+r}{-r} = -\frac{1}{r}$$

②

$$f(x) = \frac{x+r}{x^2-rx+r}, \quad g(x) = \{(r,1), (1,0), (r,0),$$

$$(0,r)\} \quad \frac{g}{f} = ?$$

$$D_g = \{0, 1, 2, 3\}$$

$$x^2 - 4x + 3 \neq 0 \Rightarrow (x-1)(x-3) \neq 0 \Rightarrow x \neq 1, 3$$

$$D_f = R - \{1, 3\}$$

$$D_{f,g} = D_g \cap D_f = \{1, 2, 3, 0\} \cap (R - \{1, 3\}) = \{0, 2\}$$

$$x=0 \Rightarrow f(0) = \frac{0+2}{0-0+3} = \frac{2}{3}, g(0) = 4$$

$$\frac{g(0)}{f(0)} = \frac{4}{\frac{2}{3}} = 4 \times \frac{3}{2} = 6 \Rightarrow (0, 6)$$

$$x=2 \Rightarrow f(2) = \frac{2+2}{4-1+3} = -1, g(2) = 1$$

$$\frac{g(2)}{f(2)} = -1 \Rightarrow (2, -1) \quad \left\{ (0, 6), (2, -1) \right\}$$

$$\textcircled{3} f(x) = \{(1, 2), (2, -1), (3, 2), (-1, 4)\}$$

$$g(x) = \{(-2, 3), (1, -1), (3, 2), (-1, 0)\}$$

$$\text{الف) } f(1/x) = \left\{ \left(\frac{1}{1}, 2\right), \left(\frac{1}{2}, -1\right), \left(\frac{1}{3}, 2\right), \left(-\frac{1}{1}, 4\right) \right\}$$

$$1/x=1 \Rightarrow x=1, f(1)=2$$

$$1/x=2 \Rightarrow x=\frac{1}{2}, f\left(\frac{1}{2}\right)=-1$$

$$2a = 2 \Rightarrow a = 1, f(1) = 2$$

$$2a = -1 \Rightarrow a = -\frac{1}{2}, f(-\frac{1}{2}) = 9$$

$$\rightarrow f(x) = \{(1, 2), (-\frac{1}{2}, 9), (2, 2), (-2, 2), (\sqrt{2}, 1), (-\sqrt{2}, 1)\}$$

$$a^2 = 1 \Rightarrow a = \pm 1, f(1) = 2$$

$$a^2 = 2 \Rightarrow a = \pm \sqrt{2}, f(\sqrt{2}) = 1$$

$$a^2 = 3 \Rightarrow a = \pm \sqrt{3}, f(\sqrt{3}) = -1$$

$$2) g^2(a) + 1 = \{(-2, 19), (-1, 3), (2, 9), (0, 1)\}$$

$$a = -2 \Rightarrow g(-2) = 3 \Rightarrow 2x^4 + 1 = 19$$

$$a = -1 \Rightarrow g(-1) = -1 \Rightarrow 2x^4 + 1 = 3$$

$$a = 2 \Rightarrow g(2) = 9 \Rightarrow 2x^4 + 1 = 9$$

$$a = -1 \Rightarrow g(-1) = 0 \Rightarrow 2x^4 + 1 = 1$$

$$3) \frac{f}{g} = \{(1, -1), (2, -1)\}$$

$$Df \cap Dg = \{1, -1, 2\} \quad g(a) \neq 0$$

$$D_{\frac{f}{g}} = \{1, 2\}$$

$$a = 1 \Rightarrow \frac{f(1)}{g(1)} = \frac{f(1)}{-1} = -1$$

$$a = 2 \Rightarrow \frac{f(2)}{g(2)} = \frac{f(-1)}{2} = -1$$

$$\textcircled{1} f(x) = \{(1, 1), (2, 1), (2, 9), (-1, 0)\}$$

$$g(x) = \{(0, 1), (2, 3), (-2, 9), (-1, 2), (3, 3)\}$$

$$h = f + g \quad \text{DF} = \{1, 2, 2, -1\}$$

$$\text{D}g = \{0, 2, -2, -1, 3\}$$

$$\text{DF} = \text{D}g \cap \text{DF} = \{1, 2, -1\}$$

$$x=1 \Rightarrow h(1) = f(1) + g(1) = 1(1) + 1 = 1+1=2$$

$$x=2 \Rightarrow h(2) = f(2) + g(2) = 1(1) + 3 = 1+3=4$$

$$x=-1 \Rightarrow h(-1) = f(-1) + g(-1) = 1(0) + 9 = 0+9=9$$

$$h = \{(1, 2), (2, 4), (-1, 9)\}$$

$$\textcircled{2} \frac{\sqrt{3x} - x}{x-1} = f(x), \quad g(x) = [x] + [-x]$$

$$\left(\frac{f}{g+1}\right)(x)$$

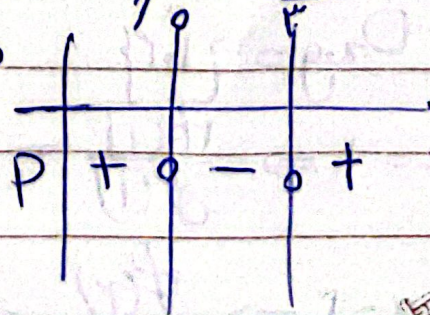
domain

$$\sqrt{3x} - x \geq 0 \Rightarrow \sqrt{3x} \geq x \Rightarrow 3x \geq x^2 \Rightarrow$$

$$x^2 - 3x \leq 0 \Rightarrow x(x-3) \leq 0$$

$$[0, 3]$$

$$x-1 \neq 0 \Rightarrow x \neq 1$$



$$D_f = [0, 4] - \{2\}$$

$$g(x) = [x] + [-x]$$

$$\Rightarrow \text{برای } x \text{ صحیح} \quad [x] + [-x] = x - x = 0$$

$$\Rightarrow \text{برای } x \text{ صحیح نباشد} \quad [x] + [-x] = -1$$

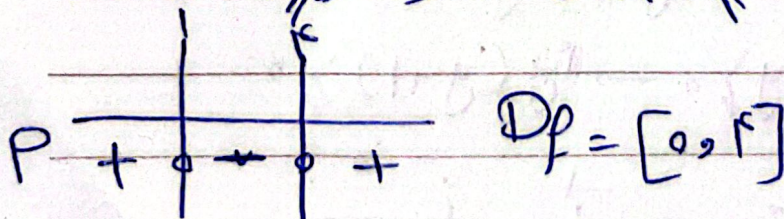
$$g(x) + 1 \neq 0 \Rightarrow \text{برای } x \text{ صحیح نباشد}$$

$$x \in \{0, 1, 3\}$$

$$\textcircled{1} f(x) = \sqrt{4x - x^2 - 4}, \quad g(x) = \{(1, 2)(2, 2)(3, 4)(4, 1)\}$$

$$h(x) = \frac{f(x)}{g(x)} = ?$$

$$4x - x^2 - 4 \geq 0 \Rightarrow x^2 - 4x + 4 \leq 0 \Rightarrow (x-2)(x-2) \leq 0$$



$$D_g = \{1, 2, 3, 4\} \quad D_h = D_f \cap D_g = \{1, 2, 3\}$$

$$\Rightarrow a=1 \quad f(1)=\sqrt{a(1)-1^2-1}=0 \quad (1,0)$$

$$\Rightarrow a=2 \quad f(2)=\sqrt{a(2)-2^2-1}=\sqrt{1} \quad (2, \frac{\sqrt{1}}{2})$$

$$\Rightarrow a=3 \quad f(3)=\sqrt{a(3)-3^2-1}=\sqrt{0}=0 \quad (3,0)$$

$$h(a)=\{(1,0), (2, \frac{\sqrt{1}}{2}), (3,0)\}$$

$$\textcircled{1} \text{ الف) } y=(a^3-3a^2+3a-1)+1$$

$$y=(a-1)^3+1 \Rightarrow (a-1)^3=y-1 \Rightarrow \sqrt[3]{y-1}=a-1$$

$$\Rightarrow a=\sqrt[3]{y-1}+1 \quad R_f=\mathbb{R}$$

$$\text{ب) } y=\frac{1}{a^2-2a} \Rightarrow y \neq 0$$

$$y(a^2-2a)=1 \Rightarrow ya^2-2ya-1=0$$

$$\Delta \geq 0 \Rightarrow 4y^2+4y \geq 0 \Rightarrow y(y+1) \geq 0$$

$$\Rightarrow y(y+1) \geq 0$$

$$R_g=(-\infty, -1] \cup (0, +\infty)$$

$$\textcircled{10} \text{ الف) } g = \frac{3x+1}{x-2}$$

$$\text{مخرج} \Rightarrow \frac{\textcircled{3}x+b}{\textcircled{1}x+d} = g \quad \frac{a}{c} = \frac{3}{1} = 3$$

$$R = \mathbb{R} - \{2\}$$

$$\text{ب) } g = \frac{2x+1}{x+1}$$

$$\text{مخرج} \Rightarrow \frac{\textcircled{2}x+b}{\textcircled{1}x+d} = g \quad \frac{a}{c} = \frac{2}{1} = 2$$

$$R = \mathbb{R} - \{1\}$$