

$$f(x) = \sqrt{x + 4\sqrt{x-4} + 4 - 4} = \sqrt{(\sqrt{x-4} + 2)^2} = \sqrt{x-4} + 2$$

$$g(x) = \sqrt{x - 4\sqrt{x-4} + 4 - 4} = \sqrt{(\sqrt{x-4} - 2)^2} = |\sqrt{x-4} - 2| \quad x-4 \geq 4 \Rightarrow x \geq 8$$

$$\sqrt{x-4} \geq 2 \rightarrow \sqrt{x-4} - 2$$

$$f(x) + g(x) = 2\sqrt{x-4} = a + b\sqrt{x+c} \rightarrow a=0, b=2, c=-4 \Rightarrow \frac{a+b}{-c} = -\frac{1}{2}$$

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$$Dg \cap Df = \{0, 1, 2, 3\} \cap \{x \in \mathbb{R} - \{1, 3\}\} = \{0, 2\}$$

$$Df = \frac{x+2}{(x+1)(x-3)} \rightarrow \mathbb{R} - \{1, 3\}$$

$$\frac{g}{f}(0) = \frac{4}{\frac{2}{3}} = 6 \quad \frac{g}{f}(2) = \frac{1}{\frac{2+3}{3}} = \frac{3}{5} \rightarrow \frac{g}{f}(x) = \{0, 6\} \cup \left(\frac{3}{5}, 1\right)$$

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الف) $\{(-\frac{1}{4}, 9), (2, 2), (\frac{3}{4}, -1), (\frac{1}{4}, 2)\}$

ب) $\{(2, 2), (-2, 2), (\sqrt{3}, -1), (-\sqrt{3}, -1), (1, 2), (-1, 2)\}$

ج) $\{(-1, 1), (3, 9), (1, 3), (-2, 1), (-1, 9)\}$

د) $\{(1, 4), (3, 1)\}$

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$$Dh = Df \cap Dg \rightarrow Dh = \{2, -2, 3\}$$

$$h(x) = 2f(x) + g(x) \rightarrow h = \{(2, 0), (-2, 4), (3, 5)\}$$

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$$f(x) \in \mathbb{Z} \rightarrow g(x) = 0 \rightarrow g(x) + 1 = 0 + 1 \rightarrow 1 \neq 0 \checkmark$$

$$f(x) \notin \mathbb{Z} \quad g(x) = -1 \rightarrow g(x) + 1 = -1 + 1 = 0 \rightarrow \text{نادرست}$$

$$Df = [0, 3] - \{2\} \leftarrow x \neq 2 \leftarrow 0 \leq x \leq 3 \leftarrow \begin{cases} 3x \geq 0 \quad x \geq 0 \\ \sqrt{3}x - x \geq 0 \\ \sqrt{3}x \geq x \end{cases}$$

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$$-x + 0x - \varepsilon \geq 0 \rightarrow x^2 - 0x + \varepsilon \leq 0 \rightarrow (x-1)(x-\varepsilon) \leq 0 \quad \frac{1}{+} \frac{1}{-} \frac{1}{+}$$

$$\{1, 2, \varepsilon\}$$

$$h(x) = \{(1, 0), (2, \sqrt{2}), (\varepsilon, 0)\}$$

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$$a) y = x^3 - 4x^2 + 3x(-1+1) = y(x-1)^3 + 1$$

$$y-1 = (x-1)^3 \rightarrow \sqrt[3]{y-1} = x-1 \rightarrow x = \sqrt[3]{y-1} + 1$$

$$R_f = \mathbb{R}$$

$$b) y = \frac{1}{x^2-2x} \rightarrow x^2y - 2xy = 1 \rightarrow x^2y - 2xy - 1 = 0$$

$$\frac{y \neq 0}{y=0} \quad x = \varepsilon y(y+1) \geq 0 \rightarrow D_f = (-\infty, -1] \cup (0, +\infty)$$

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$$a) y = x^2 - \varepsilon x + 10 \quad x = -\frac{-\varepsilon}{2} = +\frac{\varepsilon}{2}$$

$$b) -x^2 + 4x + 12 \quad x = -\frac{4}{2(-1)} = -2 \quad (-\frac{b}{2a}) \rightarrow R_f = (-\infty, 12)$$

$$c) y_{\min} = -(2)^2 + 4(2) + 12 = 12 \rightarrow [-2, +\infty) \rightarrow \sqrt{[-2, +\infty)} \rightarrow R_f = [0, +\infty)$$

$$d) 4x - x^2 \quad x = -\frac{4}{2(-1)} = 2 \rightarrow x_{\min} = -(2)^2 + 4(2) = 4 \rightarrow \sqrt{[0, 4]} \rightarrow R_f = [0, 2]$$

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$$a) \mathbb{R} \cup \emptyset$$

$$b) \mathbb{R}$$

$$c) [0, +\infty)$$

$$d) [0, +\infty)$$

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$$a) y = \frac{x^2+1}{x-2} \rightarrow D_f = \mathbb{R} - \{2\}$$

$$b) y = \frac{x^2+1}{x+1} \rightarrow D_f = \mathbb{R} - \{-1\}$$

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