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$$f(n) = \sqrt{n + \sqrt{n-1}}$$

$$g(n) = \sqrt{n - \sqrt{n-1}}$$

$$\left\{ \begin{aligned} x &= \sqrt{(n + \sqrt{n-1})(n - \sqrt{n-1})} = \sqrt{n^2 - (\sqrt{n-1})^2} \\ &= \sqrt{n^2 - (n-1)} = \sqrt{n^2 - n + 1} = \sqrt{(n-1)^2 + 1} \end{aligned} \right.$$

$$(f \circ g)^2 = f^2 \circ g^2 \circ f \circ g$$

$$\left\{ \begin{aligned} n + \sqrt{n-1} \\ n - \sqrt{n-1} \end{aligned} \right\} \circ f \circ g \circ f \rightarrow n + (n-1) = 2n-1 = (f \circ g)^2$$

$$\frac{0 \pm \sqrt{1-1}}{-1} = \frac{0}{-1} = 0$$

$$f(n) = \frac{n^2 - 1}{n^2 - 1}$$

$$g(n) = \left\{ \begin{aligned} (1, 1) \\ (1, 0) \\ (0, 1) \\ (0, 0) \end{aligned} \right\}$$

(2)

$$(n-1)(n-1) \rightarrow n \neq 1$$

$$g_f = ? \rightarrow f(n) = 0 \rightarrow n = 1$$

$$\frac{g(n)}{f(n)} = \frac{1}{1} = 1 \rightarrow \frac{1}{1} = 1$$

$$f(n) = \left\{ \begin{aligned} (1, 1) \\ (1, 0) \\ (0, 1) \\ (0, 0) \end{aligned} \right\}, g(n) = \left\{ \begin{aligned} (-1, 1) \\ (1, -1) \\ (1, 1) \\ (-1, -1) \end{aligned} \right\}$$

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$$f(n) = \left\{ \begin{aligned} (1, 1) \\ (1, 0) \\ (0, 1) \\ (0, 0) \end{aligned} \right\}$$

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$$\left\{ \begin{aligned} n^2 = 4 &\rightarrow n = \pm 2 \\ n^2 = 3 &\rightarrow n = \pm \sqrt{3} \\ n^2 = 1 &\rightarrow n = \pm 1 \end{aligned} \right.$$

$$\frac{f(n)}{g(n)} = \left\{ \begin{aligned} (1, 1) \\ (1, 0) \\ (0, 1) \\ (0, 0) \end{aligned} \right\}$$

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$$h = f \circ g \rightarrow h = \left\{ \begin{aligned} (1, 1) \\ (1, 0) \\ (0, 1) \\ (0, 0) \end{aligned} \right\}$$

$$f(n) = \sqrt{n^2 - 1}, g(n) = [n] + [-n] \quad \left(\frac{f}{g} \right)(n) = \frac{f(n)}{g(n)}$$

(5)

$$n \geq 0 \rightarrow n \geq 0$$

$$\sqrt{n^2 - 1} \geq 0 \rightarrow n(n-1) \geq 0 \rightarrow 0 \leq n \leq 1 / n-1 \neq 0 \rightarrow n \neq 1$$

$$\frac{1}{1} = 1$$

$$D_f = [0, 1] - \{1\}$$

$$[n] + [-n] = 0 \rightarrow 1 - 1 = 0$$

$$n \rightarrow 0 \rightarrow 1 - 1 = 0$$

$$\frac{f}{g} = \left\{ \begin{aligned} (0, 1) \\ (1, 1) \end{aligned} \right\}$$

$$f(n) = \sqrt{5n - n^2 - 4} \rightarrow n^2 - 5n + 4 \geq 0 \rightarrow (n-1)(n-4) \leq 0$$

$$3 \leq n \leq 4 \rightarrow [3, 4]$$

$$g(n) = \{(1, 2), (2, 2), (3, 2), (4, 2)\}$$

$$h(n) = \frac{f(n)}{g(n)} = \left\{ \left(\frac{1}{2}, \frac{1}{2} \right), \left(\frac{2}{2}, \frac{1}{2} \right), \left(\frac{3}{2}, \frac{1}{2} \right), \left(\frac{4}{2}, \frac{1}{2} \right) \right\}$$

$$\text{الف) } y = \frac{n^2 - 2n^2 + 2n}{2(n^2 - 2n + 1)} \rightarrow y(n^2 - 2n + 1) = n^2 - 2n^2 + 2n$$

$$y(n^2 - 2n + 1) = n^2 - 2n^2 + 2n \rightarrow n = \frac{y(n^2 - 2n + 1)}{1 - y}$$

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$$\text{ب) } y = \frac{1}{n^2 - 2n} \rightarrow y(n^2 - 2n) = 1$$

$$n^2 - 2n = \frac{1}{y} \rightarrow y \neq 0$$

$$y \neq \frac{1}{n^2 - 2n}$$

$$\text{الف) } y = n^2 - 2n + 1 \rightarrow \frac{-b}{2a} = \frac{2}{2} = 1 = n_{\min}$$

$$y_{\min} = 1 - 2(1) + 1 = 0$$

$$[y, +\infty)$$

$$\text{ب) } y = -n^2 + 2n + 1 \rightarrow \frac{-b}{2a} = \frac{-2}{-2} = 1$$

$$-1 + 2(1) + 1 = 2$$

$$(-\infty, 2]$$

$$2.) y = \sqrt{n^2 - 2n - 3} \rightarrow [3, +\infty)$$

$$3.) y = \sqrt{4n - n^2} \rightarrow [0, 4]$$

$$R = \sqrt{(-\infty, 4]} = [-2, 2]$$

$$\text{الف) } R \quad \text{ب) } R \quad 2.) [0, +\infty) \quad 3.) [-2, 2]$$

$$\text{الف) } y = \frac{2n+1}{n-2} \rightarrow (n-2)y = 2n+1 \rightarrow ny - 2y = 2n+1$$

$$n = \frac{1+2y}{y-2}$$

$$\text{ب) } y = \frac{2n+1}{n+1} \rightarrow y(n+1) = 2n+1$$

$$yn + y = 2n + 1$$

$$n(y-2) = 1-y \rightarrow n = \frac{1-y}{y-2}$$

$$R = \{2\}$$

(۳) x های f باید حقیقی باشند! $f(x) = \left\{ \left(\frac{1}{x}, 2 \right) \left(\frac{x}{2}, -1 \right) (2, 2) \left(-\frac{1}{x}, 4 \right) \right\}$

هرکدام از x های f را برابر x^2 قرار دهیم! $f(x^2) = \left\{ (1, 2) (-1, 2) (\sqrt{x^2}, -1) (-\sqrt{x^2}, -1) (2, 2) (-2, 2) \right\}$

این عمل روی y های و اعمال می شود: $g^2(x) + 1 = \left\{ (-2, 19) (1, 3) (3, 9) (-1, 1) \right\}$

$\frac{2f}{g} = \left\{ \left(1, \frac{2 \times 2}{-1} \right) \left(3, \frac{2 \times -1}{2} \right) \text{ و } \left(-1, \frac{2 \times 4}{-2} \right) \right\} = \left\{ (1, -4) (3, -1) \right\}$

(الف) $y = x^3 - 2x^2 + 2x - 1 + 1 = y = (x-1)^3 + 1$ (۷)

$y - 1 = (x-1)^3 \rightarrow \sqrt[3]{y-1} = x-1 \rightarrow x = \sqrt[3]{y-1} + 1$

$R_f = \mathbb{R}$

(ب) $y = \frac{1}{x^2 - 2x} \rightarrow x^2 y - 2xy = 1 \rightarrow x^2 y - 2xy - 1 = 0$

$y \neq 0 \rightarrow x = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y} \rightarrow 4y(y+1) \geq 0 \rightarrow \frac{-1}{+} \frac{0}{-} \frac{+}{+}$

$y = 0 \rightarrow 0 = 1 \times$ صحتی وجود ندارد! $\rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$

الف - ۱) $y = x^2 - 2x + 1 \rightarrow \min \begin{cases} x = \frac{2}{2} = 1 \\ y = 1 - 2 + 1 = 0 \end{cases} \rightarrow R_f = [0, +\infty)$

ب - ۱) $y = -x^2 + 4x + 3 \rightarrow \max \begin{cases} x = \frac{-4}{-2} = 2 \\ y = -9 + 16 + 3 = 10 \end{cases} \rightarrow R_f = (-\infty, 10]$

ج - ۱) $y = x^2 - 2x - 3 \rightarrow \min \begin{cases} x = \frac{2}{2} = 1 \\ y = 1 - 2 - 3 = -4 \end{cases} \rightarrow R_f = \sqrt{[-4, +\infty)} = [-4, +\infty)$

د - ۱) $y = 4x - x^2 \rightarrow \max \begin{cases} x = \frac{4}{2} = 2 \\ y = 16 - 4 = 12 \end{cases} \rightarrow R_f = \sqrt{[-4, 9]} = [-4, 9]$