

نیکو تابش

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بازدهم دختر B



$$\left. \begin{aligned} f(x) &= \sqrt{x+4}\sqrt{x-4} = \sqrt{x-4+4}\sqrt{x-4} = \sqrt{(\sqrt{x-4}+2)^2} = \sqrt{x-4}+2 \\ g(x) &= \sqrt{x-4}\sqrt{x-4} = \sqrt{x-4+4-4}\sqrt{x-4} = \sqrt{(\sqrt{x-4}-2)^2} = \sqrt{x-4}-2 \end{aligned} \right\} \Rightarrow$$

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$$f(x) + g(x) = 2\sqrt{x-4} \Rightarrow \left. \begin{aligned} a &= 0 \\ b &= 2 \\ c &= -4 \end{aligned} \right\} \Rightarrow \frac{a+b}{c} = \frac{2}{-4} = \boxed{-\frac{1}{2}}$$

$$\left. \begin{aligned} f(x) &= \frac{x+2}{x^2-4x+3} = \frac{x+2}{(x-1)(x-3)} \Rightarrow D_f = \mathbb{R} - \{1, 3\} \\ g(x) &= \{(1, 1), (1, 0), (3, 2), (0, 4)\} \end{aligned} \right\} \Rightarrow D_f \cap D_g = \{2, 0\} \Rightarrow \frac{g}{f} = \left\{ \left(2, \frac{1}{\frac{4}{2}-4+3}\right), \left(0, \frac{1}{\frac{0}{2}-4+3}\right) \right\} = \left\{ \left(2, \frac{1}{-1}\right), \left(0, \frac{1}{-3}\right) \right\} = \left\{ \left(2, -1\right), \left(0, -\frac{1}{3}\right) \right\}$$

$$\left. \begin{aligned} \text{الف)} f(2x) &= \left\{ \left(\frac{1}{2}, 2\right), \left(\frac{3}{2}, -1\right), (2, 2), \left(\frac{1}{2}, 1\right) \right\} \\ \text{ب)} f(x^2) &= \{(1, 2), (-1, 2), (\sqrt{3}, 1), (-\sqrt{3}, 1), (2, 2), (-2, 2)\} \\ \text{ج)} 2g^2(x) + 1 &= \{(-2, 1), (1, 3), (3, 9), (-1, 1)\} \end{aligned} \right\} \Rightarrow \frac{2f}{g} = \left\{ (1, -2), (3, -1), \left(-1, \frac{12}{-1}\right) \right\}$$

برای $x = -1$ هم غرق است تعریف نشده

$$h = 2f + g = \left\{ (2, 2), (3, 2), (2, 6) \right\}$$

$$\left(\frac{f}{g+1} \right)(x) = \frac{\sqrt{3x-x}}{x-2} = \frac{\sqrt{2x}}{x-2} \Rightarrow \left. \begin{aligned} 1) 3x > 0 &\Rightarrow x > 0 \\ 2) \sqrt{3x-x} > 0 &\Rightarrow \sqrt{2x} > 0 \Rightarrow x > 0 \\ 3) x-2 \neq 0 &\Rightarrow x \neq 2 \\ 4) [x] + [-x] + 1 \neq 0 &\Rightarrow [x] + [-x] \neq -1 \Rightarrow x \in \mathbb{Z} \end{aligned} \right\} \Rightarrow \frac{2}{x-2} + 1 = \frac{2+x-2}{x-2} = \frac{x}{x-2} \Rightarrow [0, 3]$$

$$D_{\frac{f}{g+1}} = \{0, 1, 3\}$$

$$\left. \begin{aligned} f(x) &= \sqrt{2x-x^2-4} = \sqrt{-(x^2-2x+4)} = \sqrt{-(x-1)^2+3} \Rightarrow \frac{1}{x-1} \Rightarrow [1, 4] = D_f \\ h(x) &= \frac{f(x)}{g(x)} \Rightarrow g(x) \neq 0 \Rightarrow D_g = \{1, 2, 4, \infty\} \\ h(x) &= \left\{ (1, 0), \left(2, \frac{\sqrt{2}}{2}\right), (4, 0) \right\} = \left\{ (1, 0), \left(2, \frac{1}{\sqrt{2}}\right), (4, 0) \right\} \end{aligned} \right\} \Rightarrow D_h = \{1, 2, 4\}$$

الف) $y = x^3 - 3x^2 + 3x = x^3 - 3x^2 + 3x - 1 + 1 = (x-1)^3 + 1 \Rightarrow y-1 = (x-1)^3 \Rightarrow x-1 = \sqrt[3]{y-1} \Rightarrow x = \sqrt[3]{y-1} + 1 \Rightarrow \boxed{R_f = \mathbb{R}}$

ب) $y = \frac{1}{x^2 - 2x} \Rightarrow x^2 y - 2xy - 1 = 0 \Rightarrow \textcircled{1} y \neq 0 \Rightarrow x = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y} \Rightarrow \frac{-1}{y(y+1)} \neq 0 \Rightarrow \frac{-1}{y(y+1)} \neq 0 \Rightarrow (-\infty, -1] \cup [0, +\infty)$

② $y = 0 \Rightarrow 0 = 1 \times \text{غلط}$

$\boxed{R_f = (-\infty, -1] \cup [0, +\infty)}$

الف) $y = x^2 - 4x + 1 \Rightarrow \text{parabola} \left| \begin{array}{l} \frac{f}{y} = 2 \\ f = 1 \end{array} \right. \xrightarrow{\text{min}} \boxed{R_f = [1, +\infty)}$

ب) $y = -x^2 + 4x + 3 \Rightarrow \text{parabola} \left| \begin{array}{l} \frac{-f}{-y} = 3 \\ -9 + 12 + 3 = 6 \end{array} \right. \xrightarrow{\text{max}} \boxed{R_f = (-\infty, 6]}$

ج) $y = \sqrt{x^2 - 4x - 3} \Rightarrow \left| \begin{array}{l} \frac{f}{y} = 2 \\ f = -11 \end{array} \right. \xrightarrow{\text{min}} \sqrt{[-7, +\infty)} \Rightarrow \boxed{R_f = [0, +\infty)}$

د) $y = \sqrt{6x - x^2} \Rightarrow \left| \begin{array}{l} \frac{-f}{-y} = 3 \\ 18 - 9 = 9 \end{array} \right. \xrightarrow{\text{max}} \sqrt{(-\infty, 9]} \Rightarrow \boxed{R_f = [0, 3]}$

الف) $y = x^3 - 6x^2 + 9x + 1 \Rightarrow \boxed{R_f = \mathbb{R}}$

ب) $y = x^3 - 2x^2 + 3x + 2 \Rightarrow \boxed{R_f = \mathbb{R}}$

ج) $y = \sqrt{x^3 - 4x^2 + 6x + 1} = \sqrt{1x} \Rightarrow \boxed{R_f = [0, +\infty)}$

د) $y = (x^5 - 4x^2 + 3x + 1)^2 \Rightarrow \text{چون عبارت به توان دو رسیده است حاصل همواره + یا صفر است} \Rightarrow \boxed{R_f = [0, +\infty)}$

الف) $y = \frac{3x+1}{x-2} \Rightarrow \boxed{R_f = \mathbb{R} - \left\{ \frac{7}{5} \right\}}$

ب) $y = \frac{2x+1}{x+1} \Rightarrow \boxed{R_f = \mathbb{R} - \left\{ \frac{2}{1} \right\}}$