

۲. آزمون عالی نوشتن

$$\Rightarrow \sqrt{t^2 + \varepsilon + \varepsilon t} + \sqrt{t^2 + \varepsilon - \varepsilon t} = a + b\sqrt{q_1 + c} \rightarrow \sqrt{(t+\varepsilon)^2} + \sqrt{(t-\varepsilon)^2} = a + b\sqrt{q_1 + c}$$

$$|t+\varepsilon| + |t-\varepsilon| = |\sqrt{q_1 - \varepsilon} + \varepsilon| + |\sqrt{q_1 - \varepsilon} - \varepsilon|$$

$$= 2\sqrt{q_1 - \varepsilon}$$

$$b = 2, a = 0, c = -\varepsilon$$

$$\frac{a+b}{c} = \frac{2}{-\varepsilon} = -\frac{2}{\varepsilon}$$

$$f(q_1) = \frac{q_1 + 2}{q_1^2 - \varepsilon q_1 + 3} \rightarrow f(q_1) = \left\{ (2, -2), (0, \frac{2}{3}) \right\}$$

$$g/f = \left\{ (2, \frac{1}{2}), (0, \frac{1}{3}) \right\}$$

الف) $f(q_1) = \left\{ (2, 2), (3, -1), (2, 2), (-1, 4) \right\}$

ب) $f(q_1) = \left\{ (1, 2), (\sqrt{3}, -1), (-1, 2), (-\sqrt{3}, -1), (2, 2), (-2, 2) \right\}$

ج) $rg^2(q_1) + 1 = \left\{ (-2, 14), (1, 3), (3, 4), (-1, 1) \right\}$

د) $\frac{rf}{g} = \left\{ (-1, -2), (1, -2), (3, -2) \right\}$

$$h = rf + g = \left\{ (2, 5), (3, 4), (3, 5) \right\}$$

$$h(q_1) = \left(\frac{f}{g+1} \right) q_1 = \frac{\sqrt{(q_1^2 - 1) - q_1}}{(q_1 - 1)([q_1 + q_1] + 1)}$$

$$\rightarrow \sqrt{q_1} \geq q_1 \rightarrow q_1 \geq q_1^2 \rightarrow q_1^2 - q_1 \leq 0$$

$$q_1 \in [0, 1]$$

$$D_h = \left\{ 0, 1, 3 \right\}$$

$$f(x) = \{(1,0), (2,0), (2, \sqrt{2})\}$$

$$f(x) = \sqrt{-(x-1)(x-2)}$$

$$h(x) = \frac{f(x)}{g(x)} = \{(1,0), (2,0), (2, \frac{\sqrt{2}}{2})\}$$

6

الف) $y = x^3 - 3x^2 + 3x \rightarrow (x-1)^3 + 1 = y \rightarrow (x-1)^3 = y-1$
 $x-1 = \sqrt[3]{y-1} \rightarrow x = \sqrt[3]{y-1} + 1$
 $R_f = \mathbb{R}$

7

ب) $y = \frac{1}{x^2 - 2x} \rightarrow y(x^2 - 2x) - 1 = 0$
 $R_f = (-\infty, -1] \cup (0, +\infty)$

$$x = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y}$$

$\Delta y(y+1) \geq 0$
 $x \in [-\infty, -1] \cup [0, +\infty)$

الف) $y = x^2 - 2x + 10 \quad a > 0 \quad x_{\min} = \frac{2}{2} = 1, y_{\min} = 9 \quad R_f = [9, +\infty)$

ب) $y = -x^2 + 4x + 3 \quad a < 0 \quad x_{\max} = \frac{4}{-2} = -2, y_{\max} = 11 \quad R_f = (-\infty, 11]$

8

ج) $y = \sqrt{x^2 - 2x - 3} \quad a > 0 \quad x_{\min} = 1, y_{\min} = -1 \quad R_f = [0, +\infty)$

د) $y = \sqrt{4x - x^2} \quad a < 0 \quad x_{\max} = 2, y_{\max} = 2 \quad R_f = [0, 2]$

الف) $y = x^3 - 2x^2 + 3x + 1 \quad R_f = \mathbb{R}$

ب) $y = x^4 - 4x^2 + 3x + 2 \quad R_f = \mathbb{R}$

ج) $y = \sqrt{x^4 - 4x^2 + 5x + 1} \quad R_f = [0, +\infty)$

د) $y = (x^5 - 2x^2 + 3x + 1)^2 \quad R_f = [0, +\infty)$

9

الف) $y = \frac{x+1}{x-2} \quad R_f = \mathbb{R} - \{2\}$

ب) $y = \frac{x+1}{x+1} \quad R_f = \mathbb{R} - \{2\}$

10