

تکلیف ۱

۲. آزمون علی بن موسی

آزمون علی بن موسی

$$\sqrt{x-4} = t \rightarrow x = t^2 + 4 \rightarrow f(x) = \sqrt{t^2 + 4 + 4t} = \sqrt{(t+2)^2}$$

$$t^2 + 4 \geq 1 \rightarrow t^2 \geq -3 \rightarrow t \geq 2 \Rightarrow f(x) = t+2 \Rightarrow g(x) = t-2$$

$$f(x) + g(x) = 4 \rightarrow 2\sqrt{x-4} \Rightarrow \frac{a}{a+b\sqrt{x+c}} \Rightarrow a=0, b=2, c=-4 \quad \frac{a+b}{c} = -\frac{2}{4}$$

$$\frac{x+2}{(x-1)(x-3)} \rightarrow x \neq 1, x \neq 3 \xrightarrow{g(x)} (2, 1), (0, 3)$$

$$\Rightarrow f(2) = \frac{2+2}{(1)(-1)} = -4 \quad g(2) = 1 \Rightarrow \frac{g(2)}{f(2)} = -\frac{1}{4}$$

$$\Rightarrow f(0) = \frac{2}{3} \Rightarrow g(0) = 3 \rightarrow \frac{3}{\frac{2}{3}} = 9 \Rightarrow \frac{g}{f} = (2, -\frac{1}{4}), (0, 9)$$

$$1, 3, 4, -1 \Rightarrow 2x=1, \dots \Rightarrow x=\frac{1}{2}, x=\frac{3}{2}, x=2, x=-\frac{1}{2}$$

$$f(2x) = \left\{ \left(\frac{1}{2}, 2\right), \left(\frac{3}{2}, 1\right), (2, 2), \left(-\frac{1}{2}, 6\right) \right\}$$

$$f(x^2) \Rightarrow x^2 \geq 0 \rightarrow (-1, 6) \Rightarrow x^2=1 \quad x^2=3 \quad x^2=4$$

$$f(x^2) = \left\{ (2, 2), (2, 2), (-1, 2), (1, 2), (-\sqrt{3}, -1), (\sqrt{3}, -1) \right\}$$

$$2g(x)^2 + 1 \Rightarrow g(2)=3 \rightarrow 2g(x)^2 + 1 \rightarrow 19$$

$$g(-1)=0 \rightarrow 1 \quad g(1)=-1 \rightarrow 3 \quad g(3)=2 \rightarrow 9$$

$$\left\{ (-2, 19), (1, 3), (3, 9), (-1, 1) \right\}$$

$$1, 3, -1 \quad \text{مشترک}$$

$$g(-1)=0 \quad 2f(1)=4 \quad g(1)=-1$$

$$2f(3)=2 \quad g(3)=2 \quad \frac{2f}{g} = (1, 3)(3, -1)$$

$$h = 2f + g$$

$$\text{مشترک} = 2, 3, -2$$

$$x=2 \rightarrow h=5$$

$$h(3)=5 \quad h(-2)=6$$

$$h = \left\{ (2, 5), (3, 5), (-2, 6) \right\}$$

5) $g(x) = [x] + [-x] \rightarrow$ مخرج $= g+1 \rightarrow g \neq -1 \rightarrow$ با هر عدد صحیح باشد $x \in \mathbb{Z}$

6) $f(x) \rightarrow x \geq 0 \rightarrow x \geq 0 \rightarrow \sqrt{x-x} \geq 0 \rightarrow \sqrt{x} \geq x \rightarrow \begin{cases} 0 \leq x \leq 1 \\ x \geq 1 \end{cases} \Rightarrow x-1 \neq 0 \Rightarrow x \neq 1$
 $D_f = [0, 1] - \{1\} \xrightarrow{\text{مجموعه}} D = [0, 1)$
 $\Delta x - x^2 - 4 \geq 0$

7) مشترکها $\rightarrow 1, 2, 4$
 $h = \{(1, 0), (2, \frac{\sqrt{2}}{2}), (4, 0)\}$
 $\begin{cases} f(1) = 0 & f(2) = \sqrt{2} & f(4) = 0 \\ h(1) = 0 & h(2) = \frac{\sqrt{2}}{2} & h(4) = 0 \end{cases}$
 $(x-1)(x-4) \leq 0 \rightarrow 1 \leq x \leq 4$
 $x^3 - 3x^2 + 3x$ (الف)

8) $t = (x-1)^3 - 1$
 $(x-1)^3 \geq 0$ چون $t \geq -1 \rightarrow t \neq 0$
 $y = \frac{1}{x^2 - 2x}$ (ب)
 $y = (x-1)^3 + 1$
 $R = \mathbb{R}$ (ج)

9) $[-1, 0) \cup (0, +\infty)$
 $R = (-\infty, 1] \cup (0, +\infty)$
 $y = \frac{1}{t}$
 $-1 < t < 0 \rightarrow y < -1$
 $t > 0 \rightarrow y > 0$
 $x = \frac{-b}{2a}$ (الف)

$y = 12$
 $x = 3 \rightarrow \frac{-b}{2a} = \frac{-6}{-2} = 3$ (ب)
 $R = (-\infty, 11]$
 $\frac{-b}{2a} = \frac{-6}{-2} = 3$ (ج)
 $R = [0, 13]$
 $\frac{-b}{2a} = \frac{6}{2} = 3 \rightarrow x = 3 \rightarrow y = 9$
 $R = [9, +\infty)$

$x = 3 \rightarrow y = \sqrt{9} \rightarrow 3$
 $R = [0, 13]$
 $\frac{-b}{2a} = \frac{6}{2} = 3 \rightarrow x = 3 \rightarrow y = 9$ (ج)
 $R = [0, +\infty)$
 $R_f = [0, +\infty)$ (ج)
 $\text{یا به رادیکال فقط می تواند یا + یا - باشد}$

10) $\mathbb{R}$ (ب)
 $\mathbb{R}$ (الف)
 $[0, +\infty)$ (د)

11) $\mathbb{R} - \{\frac{c}{a}\} \Rightarrow R_f = \mathbb{R} - \{1\}$ (ب)
 $\mathbb{R} - \{\frac{c}{a}\} \Rightarrow R_f = \mathbb{R} - \{3\}$ (الف)