

(۱)

$$f(x) = \sqrt{x + \varepsilon \sqrt{x - \varepsilon}} \quad x \geq 1$$

$$g(x) = \sqrt{x - \varepsilon \sqrt{x - \varepsilon}}$$

$$f(x) + g(x) = a + b\sqrt{x + c}$$

$$\frac{a+b}{c} = ?$$

$$\sqrt{x + \varepsilon \sqrt{x - \varepsilon}} + \sqrt{x - \varepsilon \sqrt{x - \varepsilon}} = a + b\sqrt{x + c}$$

$$\sqrt{x - \varepsilon} = t \rightarrow x - \varepsilon = t^2 \rightarrow x = t^2 + \varepsilon$$

$$\rightarrow \sqrt{t^2 + \varepsilon + \varepsilon t} + \sqrt{t^2 + \varepsilon - \varepsilon t} = \sqrt{(t + \varepsilon)^2} + \sqrt{(t - \varepsilon)^2} = |t + \varepsilon| + |t - \varepsilon|$$

$$\rightarrow t = \sqrt{x - \varepsilon} \geq \sqrt{1 - \varepsilon} = 1 \rightarrow t \geq 1 \rightarrow t + \varepsilon + t - \varepsilon = 2t = 2\sqrt{x - \varepsilon}$$

$$\rightarrow a + b\sqrt{x + c} = 2\sqrt{x - \varepsilon} \rightarrow a = 0, b = 2, c = -\varepsilon \rightarrow \frac{a+b}{c} = \frac{0+2}{-\varepsilon} = -\frac{2}{\varepsilon}$$

(۲)

$$f(x) = \frac{x+2}{x^2 - \varepsilon x + 3}$$

$$g(x) = \left\{ (2, 1), (1, 0), (3, 5), (0, 4) \right\}$$

$$\frac{g}{f} = ?$$

$$\rightarrow \frac{g}{f} = \left\{ (2, -\frac{1}{\varepsilon}), (0, 4) \right\}$$

(۳)

$$f(x) = \left\{ (1, 2), (3, -1), (2, 2), (-1, 4) \right\}$$

$$g(x) = \left\{ (-2, 3), (1, -1), (3, 2), (-1, 0) \right\}$$

$$f(2x) = \left\{ (1, 2), (3, -1), (2, 2), (-1, 4) \right\} \rightarrow$$

$$f(x^2) = \left\{ (1, 2), (3, -1), (2, 2), (-1, 4), (-\sqrt{3}, -1), (-2, 2) \right\} \rightarrow$$

$$2g^2(x) + 1 = \left\{ (-2, 19), (1, 3), (3, 5), (-1, 1) \right\} \rightarrow$$

$$\frac{2f}{g} = \left\{ (1, -\varepsilon), (3, -1) \right\} \rightarrow$$

(۴)

$$f(x) = \left\{ (2, 1), (3, 1), (2, 1), (-2, 0) \right\}$$

$$g(x) = \left\{ (5, 1), (2, 3), (-2, 4), (-1, 2), (2, 3) \right\}$$

$$h = 2f + g \rightarrow h = \left\{ (2, 5), (3, 5), (-2, 4) \right\}$$

$$f(x) = \frac{\sqrt{3x-x}}{x-2}$$

$$g(x) = [x] + [-x]$$

$$\left(\frac{f}{g+1} \right)(x) \rightarrow \mathcal{D}?$$

$$f(x) = \sqrt{2x - x^2} - 2 \rightarrow x^2 - 2x + 2 \leq 0 \rightarrow (x-1)(x-1) \leq 0 \rightarrow 1 \leq x \leq 1 \rightarrow D_f = [1, 1] \quad (4)$$

$$g(x) = \{(1, 2), (2, 1), (2, 4), (4, 1)\}$$

$$h(x) = \frac{f(x)}{g(x)} \rightarrow \{(1, 0), (2, \frac{\sqrt{2}}{2}), (4, 0)\} \quad \text{D}$$

$$\text{ا) } y = x^3 - 3x^2 + 3x \rightarrow y - 1 = (x-1)^3 \rightarrow \sqrt[3]{y-1} = x-1 \rightarrow x = \sqrt[3]{y-1} + 1 \rightarrow R_f = (-\infty, +\infty) = \mathbb{R} \quad (7)$$

$$\text{ب) } y = \frac{1}{x^2 - 2x} \rightarrow y(x^2 - 2x) = 1 \rightarrow yx^2 - 2yx - 1 = 0 \rightarrow \Delta = 4y^2 + 4y = 4y(y+1) \geq 0 \rightarrow y(y+1) \geq 0$$

نقطة صفر $\rightarrow y \leq -1$ و $y \geq 0$ $\rightarrow \frac{y \pm \sqrt{\Delta}}{2y} = x \rightarrow y \neq 0 \rightarrow y \neq 0 \rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$

$$\text{ا) } y = x^2 - 2x + 1 \rightarrow \text{ext } \begin{cases} 2 \\ 4 \end{cases} \text{ (min)} \rightarrow y \geq 4 \rightarrow R_f = [4, +\infty) \quad (1)$$

$$\text{ب) } y = -x^2 + 4x + 3 \rightarrow \text{ext } \begin{cases} 2 \\ 12 \end{cases} \text{ (max)} \rightarrow y \leq 12 \rightarrow R_f = (-\infty, 12] \quad \text{D}$$

$$\text{ج) } y = \sqrt{x^2 - 2x - 3} \rightarrow \text{ext } \begin{cases} 2 \\ -1 \end{cases} \text{ (min)} \rightarrow y \geq -1 \rightarrow y \geq 0 \rightarrow R_f = [0, +\infty) \quad \text{D}$$

$$\text{د) } y = \sqrt{4x - x^2} \rightarrow \text{ext } \begin{cases} 2 \\ 4 \end{cases} \text{ (max)} \rightarrow y \leq 4 \xrightarrow{\text{D}} 0 \leq y \leq 2 \rightarrow R_f = [0, 2] \quad \text{D}$$

$$\text{ا) } y = x^3 - 2x^2 + 2x + 1 \rightarrow R_f = (-\infty, +\infty) = \mathbb{R} \quad (9)$$

$$\text{ب) } y = x^3 - 4x^2 + 3x + 2 \rightarrow R_f = (-\infty, +\infty) = \mathbb{R}$$

$$\text{ج) } y = \sqrt{x^2 - 4x^2 + 2x + 1} \rightarrow \sqrt{(-\infty, +\infty)} \rightarrow R_f = [0, +\infty) \quad \text{D}$$

$$\text{د) } y = (x^2 - 2x^2 + 3x + 1)^2 \rightarrow ((-\infty, +\infty))^2 \rightarrow R_f = [0, +\infty) \quad \text{D}$$

$$\text{ا) } y = \frac{3x+1}{x-2} \rightarrow R_f = \mathbb{R} - \{3\} \quad (10)$$

$$\text{ب) } y = \frac{2x+1}{x+1} \rightarrow R_f = \mathbb{R} - \{2\} \quad \text{D}$$