

$$\begin{aligned}
 f(x) &= \sqrt{x+\varepsilon\sqrt{x-\varepsilon}} \quad x \geq 1 \\
 g(x) &= \sqrt{x-\varepsilon\sqrt{x-\varepsilon}} \\
 f(x)+g(x) &= a+b\sqrt{x+c} \\
 \frac{a+b}{c} &=?
 \end{aligned}
 \rightarrow
 \begin{aligned}
 \sqrt{x+\varepsilon\sqrt{x-\varepsilon}} + \sqrt{x-\varepsilon\sqrt{x-\varepsilon}} &= a+b\sqrt{x+c} \\
 \sqrt{x-\varepsilon} = t \rightarrow x-\varepsilon = t^2 \rightarrow x &= t^2 + \varepsilon \\
 \rightarrow \sqrt{t^2 + \varepsilon + \varepsilon t} + \sqrt{t^2 + \varepsilon - \varepsilon t} &= \sqrt{(t+\varepsilon)^2} + \sqrt{(t-\varepsilon)^2} = |t+\varepsilon| + |t-\varepsilon| \\
 \rightarrow t = \sqrt{x-\varepsilon} \geq \sqrt{1-\varepsilon} = 1 &\rightarrow t \geq 1 \rightarrow t+\varepsilon + t-\varepsilon = 2t = 2\sqrt{x-\varepsilon} \\
 \rightarrow a+b\sqrt{x+c} &= 2\sqrt{x-\varepsilon} \rightarrow a=0, b=2, c=-\varepsilon \Rightarrow \frac{a+b}{c} = \frac{0+2}{-\varepsilon} = -\frac{1}{\varepsilon}
 \end{aligned}
 \tag{1}$$

$$\begin{aligned}
 f(x) &= \frac{x+2}{x^2-\varepsilon x+3} \rightarrow \frac{x+2}{(x-3)(x-1)} \rightarrow x+2 \neq 0 \rightarrow x \neq -2, (x-3)(x-1) \neq 0 \rightarrow x \neq 1, x \neq 3 \\
 g(x) &= \left\{ (2,1), (1,0), (3,5), (0,4) \right\} \rightarrow \frac{g(2)}{f(2)} = \frac{1}{\frac{4}{-1+3}} = -\frac{1}{\varepsilon}, \frac{g(0)}{f(0)} = \frac{\varepsilon}{\frac{0+2}{0-0+3}} = \frac{\varepsilon}{\frac{2}{3}} = \frac{3\varepsilon}{2} \\
 \frac{g}{f} &=? \rightarrow \frac{g}{f} = \left\{ (2, -\frac{1}{\varepsilon}), (0, \frac{3\varepsilon}{2}) \right\}
 \end{aligned}
 \tag{2}$$

$$\begin{aligned}
 f(x) &= \left\{ (1,2), (3,-1), (2,2), (-1,4) \right\} \\
 g(x) &= \left\{ (-2,3), (1,-1), (3,2), (-1,0) \right\} \\
 \text{الف) } f(2x) &= \left\{ (2,2), (6,-1), (4,2), (-2,4) \right\} \Rightarrow \text{مؤلفه اول را } 2 \text{ می کنیم} \\
 \text{ب) } f(x^2) &= \left\{ (1,2), (\sqrt{3},-1), (2,2), (-1,4), (-\sqrt{3},-1), (-2,2) \right\} \rightarrow \text{علاقه } x^2=1 \text{ و } x^2=3 \text{ را در نظر می گیریم و دوم را هم } x^2=2 \\
 \text{ج) } 2g^2(x)+1 &= \left\{ (-2,19), (1,3), (3,9), (-1,1) \right\} \Rightarrow \text{مؤلفه دوم را به توان ۲ می رسانیم و دوم را هم } 2 \text{ می کنیم} \\
 \text{د) } \frac{2f}{g} &= \left\{ (1,-\varepsilon), (3,-1) \right\} \Rightarrow \text{در زوج های } f \text{ با مؤلفه های اول مشترک بین } f \text{ و } g \text{، مؤلفه دوم } f \text{ را دو برابر می کنیم و } \frac{12}{\varepsilon} = \text{تساوی}
 \end{aligned}$$

$$\begin{aligned}
 f(x) &= \left\{ (2,1), (3,1), (2,1), (-2,0) \right\} \\
 g(x) &= \left\{ (5,1), (2,3), (-2,4), (-1,2), (2,2) \right\} \\
 h &= 2f+g \rightarrow h = \left\{ (2,5), (3,5), (-2,4) \right\}
 \end{aligned}
 \tag{3}$$

$$\begin{aligned}
 f(x) &= \frac{\sqrt{3x-x}}{x-2} \rightarrow \begin{aligned} &^1 x \geq 0 \rightarrow x \geq 0 \rightarrow \sqrt{3x-x} \geq 0 \rightarrow \sqrt{3x} \geq x \rightarrow 3x \geq x^2 \rightarrow x^2-3x \leq 0 \rightarrow x(x-3) \leq 0 \\ &\rightarrow 0 \leq x \leq 3 \end{aligned} \\
 g(x) &= [x] + [-x] \rightarrow \mathcal{D}_g = \mathbb{R} \text{ (برای هر عددی)} \\
 \left(\frac{f}{g+1}\right)(x) &\rightarrow \mathcal{D}? \rightarrow \frac{f}{g+1} \neq 0 \rightarrow x \in \mathbb{Z} \rightarrow \mathcal{D} = \mathcal{D}_f \cap \mathbb{Z} = \{0, 1, 3\}
 \end{aligned}$$

$$f(x) = \sqrt{2x - x^2} - 2 \rightarrow x^2 - 2x + 2 \leq 0 \rightarrow (x-1)(x-1) \leq 0 \rightarrow 1 \leq x \leq 1 \rightarrow \mathcal{D}_f = [1, 1] \quad (4)$$

$$g(x) = \{(1, 2), (2, 1), (2, 4), (4, 1)\}$$

$$h(x) = \frac{f(x)}{g(x)} \rightarrow \{(1, 0), (2, \frac{1}{2}), (4, 0)\}$$

$$(a) y = x^3 - 3x^2 + 3x \rightarrow y - 1 = (x-1)^3 \rightarrow \sqrt[3]{y-1} = x-1 \rightarrow x = \sqrt[3]{y-1} + 1 \rightarrow \mathcal{R}_f = (-\infty, +\infty) = \mathcal{R} \quad (7)$$

$$(b) y = \frac{1}{x^2 - 2x} \rightarrow y(x^2 - 2x) = 1 \rightarrow yx^2 - 2yx - 1 = 0 \rightarrow \Delta = 4y^2 + 4y = 4y(y+1) \geq 0 \rightarrow y(y+1) \geq 0$$

$$\text{نقطة صفر} \rightarrow y \leq -1 \vee y \geq 0 \rightarrow \frac{2y \pm \sqrt{\Delta}}{2y} = x \rightarrow 2y \neq 0 \rightarrow y \neq 0 \rightarrow \mathcal{R}_f = (-\infty, -1] \cup (0, +\infty)$$

$$(a) y = x^2 - 2x + 1 \rightarrow \text{ext} \begin{cases} 2 \\ 4 \end{cases} \text{ (min)} \rightarrow y \geq 4 \rightarrow \mathcal{R}_f = [4, +\infty) \quad (1)$$

$$(b) y = -x^2 + 4x + 3 \rightarrow \text{ext} \begin{cases} 2 \\ 12 \end{cases} \text{ (max)} \rightarrow y \leq 12 \rightarrow \mathcal{R}_f = (-\infty, 12]$$

$$(c) y = \sqrt{x^2 - 2x - 3} \rightarrow \text{ext} \begin{cases} 2 \\ -1 \end{cases} \text{ (min)} \rightarrow y \geq -\sqrt{\frac{1}{4}} \rightarrow y \geq 0 \rightarrow \mathcal{R}_f = [0, +\infty)$$

$$(d) y = \sqrt{4x - x^2} \rightarrow \text{ext} \begin{cases} 2 \\ 4 \end{cases} \text{ (max)} \rightarrow y \leq 2 \xrightarrow{\text{نقطة صفر}} 0 \leq y \leq 2 \rightarrow \mathcal{R}_f = [0, 2]$$

$$(a) y = x^3 - 2x^2 + 2x + 1 \rightarrow \mathcal{R}_f = (-\infty, +\infty) = \mathcal{R} \quad (9)$$

$$(b) y = x^3 - 4x^2 + 3x + 2 \rightarrow \mathcal{R}_f = (-\infty, +\infty) = \mathcal{R}$$

$$(c) y = \sqrt{x^3 - 4x^2 + 2x + 1} \rightarrow \sqrt{(-\infty, +\infty)} \rightarrow \mathcal{R}_f = [0, +\infty)$$

$$(d) y = (x^3 - 2x^2 + 3x + 1)^2 \rightarrow ((-\infty, +\infty))^2 \rightarrow \mathcal{R}_f = [0, +\infty)$$

$$(a) y = \frac{3x+1}{x-2} \rightarrow \mathcal{R}_f = \mathcal{R} - \{2\} \quad (10)$$

$$(b) y = \frac{2x+1}{x+1} \rightarrow \mathcal{R}_f = \mathcal{R} - \{1\}$$