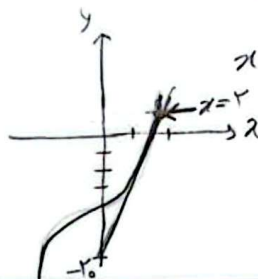


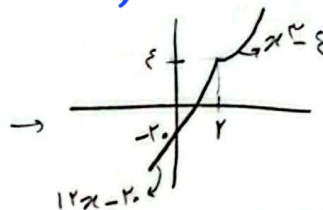
$$f(x) = \begin{cases} x^3 - \varepsilon & x \geq a \\ 12x - 20 & x \leq a \end{cases}$$



$$\begin{aligned} x^3 - \varepsilon x &= 12x - 20 \\ x^3 - 12x + 14 &= 0 \\ x &= 2 \end{aligned}$$

$$a = 2 \rightarrow \begin{cases} x^3 - \varepsilon & ; x \geq 2 \\ 12x - 20 & ; x \leq 2 \end{cases}$$

$$\begin{aligned} a^3 - 12a &\geq 12a - 20 \\ a &\in [-1, +\infty) \end{aligned}$$



الف) $f(1) = 11 - 10 = 1$

$$f^{-1}(2) = \varepsilon \rightarrow f(\varepsilon) = 2 \rightarrow 12\varepsilon - 20 = 2 \rightarrow \varepsilon = 1$$

ب) $f(f(x)) = 3(12x - 20) - 10 = 36x - 70$

$$A(2a, a) \rightarrow A'(a, 2a)$$

$$2a = \frac{a^2}{a-1} \rightarrow 2a^2 - 2a = a^2$$

$$a^2 - 2a = 0$$

$$a < 0 \quad \times$$

$$a > 0 \quad \checkmark$$

الف) $\begin{aligned} 3 &\rightarrow 5 \rightarrow x \\ \varepsilon &\rightarrow 1 \rightarrow 2 \\ 1 &\rightarrow 2 \rightarrow x \\ 9 &\rightarrow 4 \rightarrow x \end{aligned} \rightarrow f \circ f^{-1} = \{(5, 3)\}$

ب) $f^{-1} \circ f(x) = \{(3, 3), (5, 5), (1, 1), (9, 9)\}$

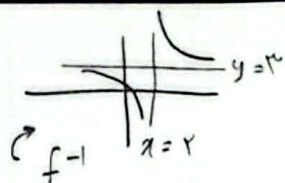
ج) $f \circ g^{-1}(x) = \{(2, 0), (5, 4)\}$

د) $g^{-1} \circ f = \{(3, 4), (5, 3), (9, 8)\}$

$$Df = \{2, 4, 9\} \quad Dg^{-1} = \{1, 3, 5\} \quad Dh = \{1, 3, 5, 9\}$$

$$\frac{h(3)}{f(g^{-1}(3))} = \frac{5}{1} = 5$$

$$\hookrightarrow \{(1, \frac{1}{2}), (3, \frac{1}{3}), (5, \frac{1}{5})\}$$



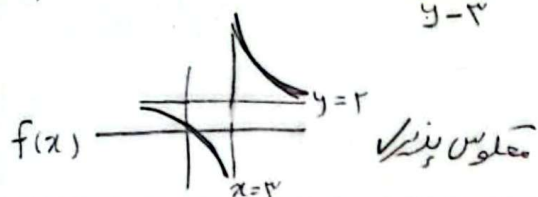
$$yx - 2y = 2x + 1$$

$$x(y - 2) = 1 + 2y$$

$$x = \frac{1 + 2y}{y - 2} \rightarrow f^{-1}(x) = \frac{1 + 2x}{x - 2}$$

5

6



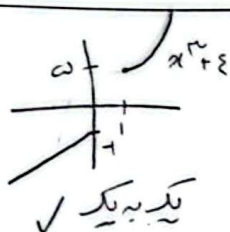
$$\Rightarrow [a, b] = [1, 3]$$

$$f(x) = 2x - 1, 1 \leq x \leq 3$$

$$f^{-1}(x) = \frac{x+1}{2}; 1 \leq x \leq 5$$

1, 5

7



$$x = y^3 + 1 \rightarrow y = \sqrt[3]{x-1} \geq 1 \rightarrow x \geq 8$$

$$x = 1 - y \rightarrow y = \frac{x+1}{-1} \leq 0 \rightarrow x \leq -1$$

$$f^{-1}(x) = \begin{cases} \sqrt[3]{x-1} & ; x \geq 8 \\ \frac{x+1}{-1} & ; x \leq -1 \end{cases}$$

5

8

$$y = \frac{x^3 + 3x^2 - x^3 - 3x^2 - 3x - 1}{x + 3} = \frac{-3x - 1}{x + 3} \rightarrow yx + 3y = -3x - 1$$

$$x(y + 3) = -3y - 1$$

$$f^{-1}(x) = \frac{-3x - 1}{x + 3} = \frac{-3x - 1}{x + 3}$$

$$f^{-1}\left(\frac{-1}{2}\right) = \frac{(-4x - 2) - 2}{(3x - 2) + 4} = \frac{-4x - 4}{3x + 2} = \frac{-4}{3} = \Delta$$

$$f^{-1}(b) = \Delta$$

5

9

$$x = \frac{y}{y^2 + 1} \rightarrow xy^2 + x = y \rightarrow xy^2 - y + x = 0 \rightarrow y = \frac{1 \pm \sqrt{1 - 4x^2}}{2x}$$

$$\mathbb{D} f \rightarrow \mathbb{R}, \mathbb{R} g \rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2}$$

$$\Rightarrow f^{-1}(x) = \frac{1 + \sqrt{1 - 4x^2}}{2x}$$

5

10