

$$f(x) = \begin{cases} x^3 - 4 & x > a \\ 12x - 10 & x \leq a \end{cases} \quad \begin{aligned} &\rightarrow 12a - 10 \leq a^3 - 4 \\ &-a^3 + 12a - 14 \leq 0 \end{aligned}$$

$$\begin{array}{c|ccc} x & -4 & 4 & \\ \hline P & - & + & + \end{array} \quad \begin{aligned} &\leftarrow (a-2)^2(a-4) \geq 0 \\ &\leftarrow a^3 - 12a + 14 \geq 0 \end{aligned}$$

پس $a \in (-\infty, -2] \cup [4, \infty)$

الف)

$$f(x) \rightarrow x = 3y + K$$

$$3y = x - K \rightarrow y = \frac{x - K}{3}$$

$$f(1) = \frac{1 - K}{3} = 4 \rightarrow K = 1$$

$$f(1) = 11 - 10 = 1$$

$$f \circ f(x) = 3(3x - 10) - 10 = 9x - 40$$

۲

$$f(x) \Rightarrow x = \frac{ay}{y-1} \rightarrow xy - x = ay \rightarrow y = \frac{x}{x-a}$$

$$a^2 - 2a = 0$$

$$a(a-2) = 0$$

$$\rightarrow a = 0$$

$$\rightarrow a = 2$$

$$a = \frac{2a}{\frac{2a}{a}}$$

$$\leftarrow A(2a, a) \leftarrow$$

۳

الف) $f \circ f^{-1} \{ (0, 0), (1, 1), (2, 2), (3, 3) \}$

ب) $f^{-1} \circ f \{ (1, 1), (2, 2), (3, 3), (4, 4) \}$

ج) $f \circ g^{-1} \{ (1, 0), (0, 1) \}$

د) $g^{-1} \circ f \{ (1, 1), (2, 2), (3, 3), (4, 4) \}$

۴

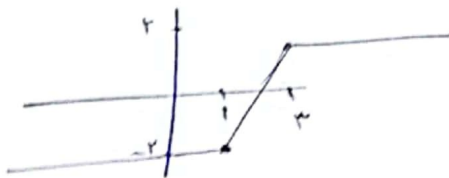
$$f \circ g^{-1} = \{ (1, 1), (2, 2), (3, 3) \}$$

$$h = \{ (1, 1), (2, 2), (3, 3), (4, 4) \}$$

$$\frac{h}{f \circ g^{-1}} = \{ (1, 1), (2, 2) \}$$

۵

$$f(x) = |x-1| - |x-3|$$



$$[0, b] = [1, 3]$$

! ۷ سوال

تابع معکوس

$$f^{-1}(x) \Rightarrow x = \frac{y+1}{y-2}$$

$$y = \frac{x+1}{x-2}$$

$$xy - 2y = x + 1$$

$$xy - 0y = 1$$

$$y(x - 0) = 1 \rightarrow y = \frac{1}{x-0}$$

! ۴ سوال

$$f(x) = \begin{cases} x^2 + 4 & x \geq 1 \\ 4x - 1 & x < 1 \end{cases}$$

$$x = y^2 + 4 \rightarrow y = \sqrt{x-4}$$

$$x = 4y + 1$$

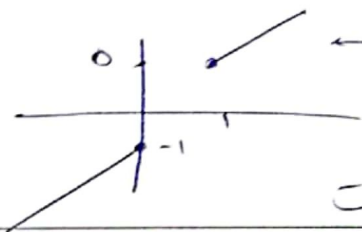
$$y = \frac{x+1}{4}$$

$$f^{-1}(x)$$

$$\sqrt{x-4} \quad x \geq 4$$

$$\frac{x+1}{4} \quad x < -1$$

۱



برای تابع $f(x)$ با دامنه
تابع $f^{-1}(x)$ برابر است

۹

$$x = \frac{y}{y^2 + 1} \rightarrow xy^2 + x = y$$

$$xy^2 + x - y = 0 \rightarrow \Delta = 1 - 4x^2$$

برای اینکه تابع معکوس
دارد باید $1 - 4x^2 \geq 0 \rightarrow 1 \geq 4x^2$

$$y = \frac{1 \pm \sqrt{1 - 4x^2}}{2x}$$

$$f^{-1}(x) = \begin{cases} \frac{1 - \sqrt{1 - 4x^2}}{2x} & x < \frac{1}{2} \\ \frac{1 + \sqrt{1 - 4x^2}}{2x} & x > \frac{1}{2} \end{cases}$$

$$\frac{1 + \sqrt{1 - 4x^2}}{2x} \quad x > \frac{1}{2}$$

۱۰