

$$f(x) = \begin{cases} x^3 - 8; x > a \rightarrow \text{روی ضربا و باقی} \\ 14x - 20; x \leq a \rightarrow \text{یک به یک} \end{cases}$$

$$x^3 - 8 = 14x - 20$$

$$x^3 - 14x + 12 = 0$$

$$(a-2)(a+4) \geq 0$$

$$\begin{array}{r} -8 \quad 2 \\ - \quad 1 \quad 1 \quad + \end{array}$$

$$a \geq -4$$

5

$$f^{-1}(y) = 8, f(x) = 3x + k \rightarrow D_{f^{-1}} = R_f, R_{f^{-1}} = D_f \Rightarrow \psi(8) + k = 2 \rightarrow k = -10$$

$$f(x) = 3x - 10$$

$$\text{الف } f(v) = \psi(v) - 10 = 11$$

$$\text{ب } f(f(x)) = \psi(3x - 10) - 10 = 9x - 40$$

3

$$f(x) = \frac{ax}{x-1}, A(2a, a) \text{ درون } f(x) \text{ دارد}$$

$$D_{f^{-1}} = R_f, R_{f^{-1}} = D_f$$

$$\frac{a^2}{a-1} = 2a \rightarrow a^2 = 2a^2 - 2a$$

$$a^2 - 2a = 0 \rightarrow \begin{cases} a = 0 \\ a = 2 \end{cases}$$

غیر قابل قبول چون در اولی
a=0 جمع ثابت می شود و
در تیرک یک به یک نسبت

5

$$f = \{(\omega, \omega), (v, v), (u, u), (q, q)\} \quad g = \{(\omega, \omega), (v, v), (u, u), (q, q)\}$$

$$f^{-1} = \{(\omega, \omega), (v, v), (u, u), (q, q)\}$$

$$g^{-1} = \{(\omega, \omega), (v, v), (u, u), (q, q)\}$$

$$\text{ب } f^{-1} \circ f = \{(\omega, \omega), (v, v), (u, u), (q, q)\}$$

$$\omega \rightarrow \omega \rightarrow \omega$$

$$\varepsilon \rightarrow v \rightarrow \varepsilon$$

$$v \rightarrow u \rightarrow v$$

$$q \rightarrow q \rightarrow q$$

$$\omega \rightarrow \omega \rightarrow \omega$$

$$v \rightarrow \varepsilon \rightarrow v$$

$$u \rightarrow v \rightarrow u$$

$$q \rightarrow q \rightarrow q$$

$$\text{ج } f \circ g^{-1} = \{(\omega, \omega), (\omega, \varepsilon)\}$$

$$\omega \rightarrow \omega \rightarrow \omega$$

$$\varepsilon \rightarrow \omega \rightarrow \varepsilon$$

$$\omega \rightarrow \varepsilon \rightarrow \omega$$

$$\text{د } g^{-1} \circ f = \{(\omega, \omega), (v, v), (u, u), (q, q)\}$$

$$\omega \rightarrow \omega \rightarrow \omega$$

$$\varepsilon \rightarrow v \rightarrow \varepsilon$$

$$v \rightarrow u \rightarrow v$$

$$q \rightarrow q \rightarrow q$$

5

$$f = \{(\omega, \omega), (v, v), (u, u), (q, q)\} \quad g = \{(\omega, \omega), (v, v), (u, u), (q, q)\} \quad h = \{(\omega, \omega), (v, v), (u, u), (q, q)\} \quad \frac{h}{f \circ g^{-1}} = ?$$

$$g^{-1} = \{(\omega, \omega), (v, v), (u, u), (q, q)\} \quad f \circ g^{-1} = \{(\omega, \omega), (v, v), (u, u), (q, q)\}$$

$$\begin{array}{l} \omega \rightarrow \omega \rightarrow \omega \\ \varepsilon \rightarrow v \rightarrow \varepsilon \\ u \rightarrow v \rightarrow u \\ \omega \rightarrow \varepsilon \rightarrow \omega \end{array}$$

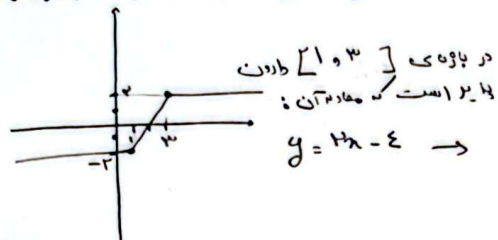
$$D_{h \cap f \circ g^{-1}} = \{(\omega, \omega), (\omega, \varepsilon)\}$$

$$\frac{h}{f \circ g^{-1}} = \left\{ \left(\omega, \frac{\omega}{\varepsilon} \right), \left(\varepsilon, \frac{\varepsilon}{\omega} \right), \left(\frac{\omega}{\varepsilon}, \frac{\varepsilon}{\omega} \right) \right\} = \left\{ \left(\omega, \frac{\omega}{\varepsilon} \right), \left(\varepsilon, \frac{\varepsilon}{\omega} \right) \right\} = \left\{ \left(\omega, \frac{\omega}{\varepsilon} \right), \left(\varepsilon, \frac{\varepsilon}{\omega} \right) \right\}$$

5

$y = \frac{x+1}{x-2}$
 در سمت راست $x=2$ و در سمت چپ $x=2$
 $x=1 \rightarrow y=-\frac{1}{2}$
 $y = \frac{x+1}{x-2} \rightarrow 2x+1 = yx-2y \rightarrow x(y-2) = yx-1 \rightarrow x = \frac{yx+1}{y-2} \rightarrow f^{-1}(x) = \frac{yx+1}{x-2}$
 در سمت راست $x=2$
 در سمت چپ $x=2$
 $x=0 \rightarrow y = \frac{1}{-2}$

$f(x) = |x-1| - |x-3|, [a, b]$



در بازه $[1, 3]$ و در بازه $[1, 3]$

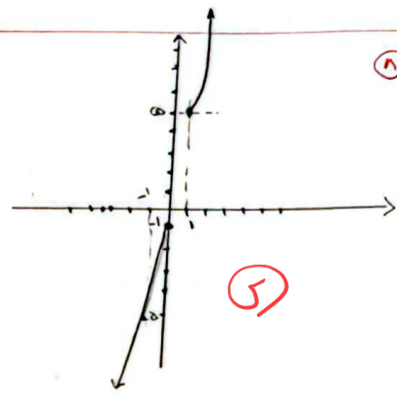
$y = 2x - 2 \rightarrow x = \frac{y+2}{2} \rightarrow y = \frac{x+2}{2} \rightarrow f^{-1}(x) = \frac{x+2}{2}$

$f(x) = \begin{cases} x^2 + 2 & x > 1 \\ x-1 & x \leq 1 \end{cases} \rightarrow R: [\omega, +\infty)$
 $f(x) = \begin{cases} x^2 + 2 & x > 1 \\ x-1 & x \leq 1 \end{cases} \rightarrow R: (-\infty, -1]$

$y = x^2 + 2 \rightarrow x = \sqrt{y-2} \rightarrow f^{-1}(x) = \sqrt{x-2}$

$y = x-1 \rightarrow x = \frac{y+1}{1} \rightarrow f^{-1}(x) = \frac{x+1}{1}$

$f^{-1}(x) = \begin{cases} \sqrt{x-2} & x \geq 2 \\ x+1 & x \leq -1 \end{cases}$



$f(x) = x^2 - \frac{(x+1)^2}{x+2}, f^{-1}(x) = \frac{a+b}{x+d}, f^{-1}(b) = ?$

$x^2 - \frac{(x+1)^2}{x+2} = \frac{x^2 + 2x^2 - x^2 - 2x - 2x - 1}{x+2} = \frac{-x^2 - 4x - 1}{x+2} \rightarrow f(x) = \frac{-x^2 - 4x - 1}{x+2}$

$y = \frac{-x^2 - 4x - 1}{x+2} \rightarrow yx + 2y = -x^2 - 4x - 1 \rightarrow x(x+4) = -1 - 2y \rightarrow x = \frac{-1 - 2y}{y+4} \rightarrow f^{-1}(x) = \frac{-x-1}{x+4}$

$\frac{-x-1}{x+4} = \frac{a+b}{x+d} \rightarrow \begin{cases} a = -1 \\ b = -1 \\ d = 4 \end{cases} \quad f^{-1}(b) = \frac{-(-1)-1}{(-1)+4} = \frac{0}{3} = 0$

$f(x) = \frac{x}{x^2+1} \rightarrow y = \frac{x}{x^2+1} \rightarrow yx^2 - x + y = 0 \rightarrow [-\frac{1}{2}, \frac{1}{2}]$

$\begin{cases} yx^2 - x + y = 0 \\ \Delta = 1 - 4y^2 \geq 0 \end{cases} \Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y} \rightarrow y \neq 0 \rightarrow y = \frac{1 \pm \sqrt{1-4x^2}}{2x} \rightarrow x \neq 0 \rightarrow f^{-1}(y) = \frac{1 \pm \sqrt{1-4x^2}}{2x}; x \in [-\frac{1}{2}, \frac{1}{2}]$

نقطه y و x هم علامت
 سب فقط $\oplus$ و $\ominus$ فقط $\oplus$