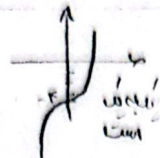


$f(x) = \begin{cases} x^3 - 4 & x > a \\ 12x - 20 & x \leq a \end{cases} \Rightarrow$

 $a > 4 \quad a^3 - 12a + 14 = (-4)^3 + 14 = -62$

$\Rightarrow \begin{cases} f(a) = a^3 - 4 \\ f(a) = 12a - 20 \end{cases} \Rightarrow a^3 - 4 = 12a - 20 \Rightarrow a^3 - 12a + 14 = 0 \Rightarrow (a-2)(a^2 + 12a - 14) = 0$
 $a = 2 \quad \Delta = 144 + 56 = 200 \quad a = \frac{-12 \pm \sqrt{200}}{2} = \frac{-12 \pm 10\sqrt{2}}{2} = -6 \pm 5\sqrt{2}$

$f^{-1}(1) = 4 \quad f(x) = 3x + k \Rightarrow f(4) = 1 = 3(4) + k = 12 + k = 1 \Rightarrow k = -10$ (الف)

$f(v) = 3v - 10 = 11$

$f(f(x)) = 3(3x - 10) - 10 = 9x - 30 - 10 = 9x - 40$ (ب)

$f(x) = \frac{ax}{x-1}$ دایره وارون تابع $A(2, a)$

$\Rightarrow ax = gx - y \Rightarrow gx - ax = y \Rightarrow x(g-a) = y \Rightarrow x = \frac{y}{g-a} \Rightarrow g = \frac{x}{x-a} \Rightarrow f^{-1}(x) = \frac{x}{x-a}$ (ج)

$\Rightarrow a = \frac{2a}{2a-a} = \frac{2a}{a} = 2 \quad a = 2$

$f = \{(3, 2), (4, 1), (5, 4), (6, 9)\}$

$g = \{(3, 2), (4, 1), (5, 4), (6, 9)\}$

$f^{-1} = \{(2, 3), (1, 4), (4, 5), (9, 6)\}$

$g^{-1} = \{(2, 3), (1, 4), (4, 5), (9, 6)\}$

الف) $f \circ f^{-1} = f(f^{-1}(x)) = \{(2, 2), (1, 1), (4, 4), (9, 9)\}$

ب) $f^{-1} \circ f = f^{-1}(f(x)) = \{(3, 3), (4, 4), (5, 5), (6, 6)\}$

ج) $f \circ g^{-1} = f(g^{-1}(x)) = \{(2, 2), (1, 1), (4, 4), (9, 9)\}$

د) $g^{-1} \circ f = g^{-1}(f(x)) = \{(3, 3), (4, 4), (5, 5), (6, 6)\}$

$f = \{(2, 4), (4, 8), (6, 16)\}$

$g = \{(2, 4), (4, 8), (6, 16)\}$

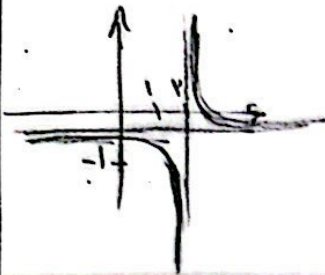
$h = \{(1, 2), (3, 4), (5, 6)\}$

$\frac{h}{f \circ g^{-1}} = \{(1, \frac{1}{2}), (\frac{3}{2}, \frac{1}{4}), (\frac{5}{2}, \frac{1}{6})\}$

$g^{-1} = \{(2, 4), (4, 8), (6, 16)\}$

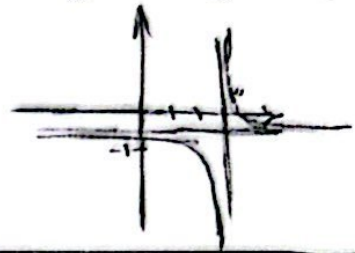
$f \circ g^{-1} = f(g^{-1}(x)) = \{(2, 4), (4, 8), (6, 16)\}$

$$y = \frac{3x+1}{x-2} \rightarrow -\frac{1}{x-2}$$



تابع هذو رافعت و پست و وارون بدلاست.

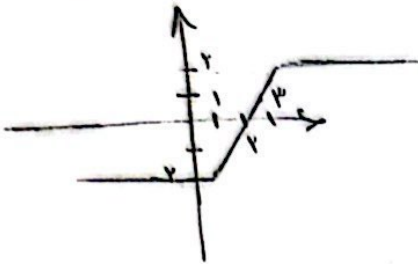
$$y = \frac{3x+1}{x-2} \rightarrow yx - 2y = 3x+1 \rightarrow yx - 3x = 2y+1 \rightarrow x(y-3) = 2y+1 \rightarrow x = \frac{2y+1}{y-3}$$



فوق و پست و وارون بدلاست

$$f(x) = |x-1| - |x-3|$$

$$[a, b] = [1, 3]$$



$$f(x) = 2x - 4 \rightarrow 2x = y + 4 \rightarrow x = \frac{y+4}{2} \rightarrow f^{-1}(x) = \frac{x+4}{2}$$

$$f(x) = \begin{cases} x^2 + 4 & x \geq 1 \\ 4x - 1 & x \leq 0 \end{cases}$$

پست و پست و وارون بدلاست.

$$y = x^2 + 4 \rightarrow x^2 = y - 4 \rightarrow x = \sqrt{y-4} \rightarrow f^{-1}(x) = \sqrt{x-4} \quad x \geq 4$$

$$y = 4x - 1 \rightarrow 4x = y + 1 \rightarrow x = \frac{y+1}{4} \rightarrow f^{-1}(x) = \frac{x+1}{4} \quad x \leq -1$$

$$f(x) = x^2 - \frac{(x+1)^2}{x+3} \rightarrow f^{-1}(x) = \frac{ax+b}{cx+d}$$

$$f(b) = ? \quad (b < 0) \quad f(-1) = +\infty$$

$$y = x^2 - \frac{(x+1)^2}{x+3} \rightarrow y(x+3) = x^2(x+3) - (x+1)^2 \rightarrow x^3 + 3x^2 - x^2 - 2x - 1 = 0 \rightarrow x^3 + 2x^2 - 2x - 1 = 0$$

$$\rightarrow x^3 + 2x^2 - 2x - 1 = 0 \rightarrow x^3 + 2x^2 - 2x - 1 = 0 \rightarrow x^3 + 2x^2 - 2x - 1 = 0 \rightarrow x^3 + 2x^2 - 2x - 1 = 0$$

$$f(x) = \frac{x}{x^2+1}$$

$$x = \frac{1 \pm \sqrt{1-4y^2}}{2y} \quad y \neq 0$$

$$y = \frac{x}{x^2+1} \rightarrow yx^2 + y = x \rightarrow yx^2 - x + y = 0$$

$$f^{-1}(x) = \frac{1 \pm \sqrt{1-4x^2}}{2x} \quad x \neq 0$$