

$$f(x) = \begin{cases} x^2 - 2; x > 1 \rightarrow R_m \Rightarrow x = 1 \rightarrow (1^2 - 2, +\infty) \\ 12x - 2; x \leq 1 \rightarrow R_m \Rightarrow x = 1 \rightarrow (-\infty, 12 \cdot 1 - 2] \end{cases} \quad 12x - 2 \leq x^2 - 2$$

$$x^2 - 12x + 14 \geq 0 \rightarrow x = 1 \rightarrow x^2 - 12x + 14 \geq 0 \rightarrow \text{پایه های معادله}$$

$$(x-1)(x+4)(x-2) \geq 0 \rightarrow \frac{-1}{1} + \frac{4}{1} + \frac{2}{1} \rightarrow x \in [-1, +\infty)$$

الف) $f(v) = \rightarrow f(1) = 1$
 $x = 1, y = 1 \rightarrow 12 + 1 = 1 \rightarrow 12 = 0$
 $f(x) = 12x - 1 \rightarrow f(v), 12 \times v - 1$
 $f(v) = 11$

$\rightarrow f(f(x)) \rightarrow f(x) = 12x - 1$
 $12 \times (12x - 1) - 1 = 144x - 12 - 1 = 144x - 13$
 $144x - 13 = f(f(x))$

$A = (12a, a) \quad f(x) = \frac{12ax}{12a - x} = y \rightarrow x = \frac{y}{12 - y} \rightarrow f^{-1}(x) = \frac{x}{12 - x}$
 $\rightarrow x = 12a, y = a \rightarrow a = \frac{12a}{12 - a} \Rightarrow a^2 = 12a \rightarrow a^2 - 12a = 0 \rightarrow a = 0 \rightarrow 12$
 $a = 0 \rightarrow f(x) = 0 \rightarrow \text{مستقیم } y = 0$
 $a = 12 \checkmark$

الف) $f \circ f^{-1} \rightarrow f^{-1} = \{(0, 12), (1, 11), (2, 10), (3, 9)\}$
 $a \rightarrow 12 \rightarrow a$
 $v \rightarrow 11 \rightarrow v$
 $u \rightarrow 10 \rightarrow u$
 $4 \rightarrow 9 \rightarrow 4$
 $f \circ f^{-1} = \{(12, 0), (11, 1), (10, 2), (9, 3)\}$

ب) $f^{-1} \circ f \rightarrow 12 \rightarrow a \rightarrow 12$
 $12 \rightarrow v \rightarrow 12$
 $v \rightarrow 11 \rightarrow v$
 $9 \rightarrow 4 \rightarrow 9$
 $f^{-1} \circ f = \{(12, 12), (11, 11), (10, 10), (9, 9)\}$

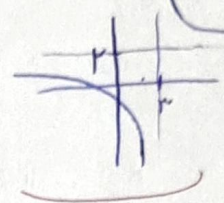
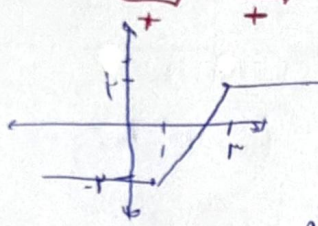
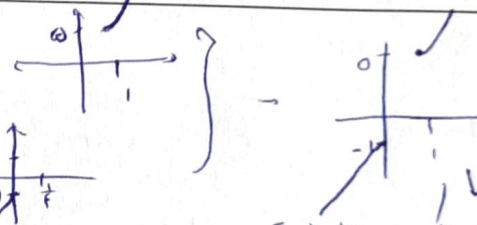
ج) $f \circ g^{-1} \rightarrow g^{-1} = \{(12, 1), (11, 2), (10, 3)\}$
 $12 \rightarrow 1 \rightarrow a$
 $11 \rightarrow 2 \rightarrow a$
 $10 \rightarrow 3 \rightarrow a$
 $f \circ g^{-1} = \{(1, 12), (2, 11), (3, 10)\}$

د) $g^{-1} \circ f \rightarrow 12 \rightarrow a \rightarrow 12$
 $12 \rightarrow v \rightarrow a$
 $v \rightarrow 11 \rightarrow 12$
 $9 \rightarrow 4 \rightarrow 11$
 $g^{-1} \circ f = \{(12, 12), (11, 11), (10, 10)\}$

$f = \{(1, 12), (2, 11), (3, 10)\}$ $g = \{(1, 1), (2, 2), (3, 3)\}$ $h = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

$f \circ g^{-1} = g^{-1} = \{(1, 12), (2, 11), (3, 10)\}$ $1 \rightarrow 1 \rightarrow 12$
 $2 \rightarrow 2 \rightarrow 11$
 $3 \rightarrow 3 \rightarrow 10$
 $h = \{(1, 1), (2, 2), (3, 3)\}$

$\frac{h}{f \circ g^{-1}} = \{(1, \frac{1}{12}), (2, \frac{1}{11}), (3, \frac{1}{10})\} = \{(1, \frac{1}{12}), (2, \frac{1}{11})\}$

$y = \frac{r\alpha + 1}{\alpha - r} \rightarrow \text{تحويل} \rightarrow \alpha = \frac{-ry - 1}{y - r} \text{ و } \frac{ry + 1}{y - r} \rightarrow f^{-1}(\alpha) = \frac{r\alpha + 1}{\alpha - r}$ بما أن $\alpha - r > 0 \rightarrow \alpha > r$ فإن $\frac{q}{c} > r$ 	6
$f(\alpha) = \frac{ \alpha - 1 }{\alpha - r} \rightarrow \text{دالة} \rightarrow \alpha - 1 > 0 \rightarrow \alpha > 1 \rightarrow y > -r$ $\alpha - r > 0 \rightarrow \alpha > r \rightarrow y > r$ $\rightarrow [a, b] = [1, r] \rightarrow 1 \leq \alpha \leq r \rightarrow \text{نقطة} \rightarrow \frac{1}{r}$ $\alpha - 1 + \alpha r > r\alpha - r \rightarrow y > r\alpha - r$ $\alpha - ry - r > ry < \alpha + r \rightarrow y < \frac{\alpha + r}{r} \rightarrow f^{-1}(\alpha) = \frac{\alpha}{r} + r$ 	7
$f(\alpha) = \begin{cases} r\alpha + r & ; \alpha \geq 1 \\ \alpha - 1 & ; \alpha < 1 \end{cases} \rightarrow \alpha = 1 \rightarrow y = \omega$ $\alpha = 0 \rightarrow y = -1$ $y < r\alpha + r \rightarrow \alpha < y/r + r \rightarrow y/r < \alpha - r \rightarrow y < r(\alpha - r)$ $y < r\alpha - 1 \rightarrow \alpha < (y + 1)/r \rightarrow ry < \alpha + 1 \rightarrow y < \frac{\alpha + 1}{r}$ $f^{-1}(\alpha) = \begin{cases} \sqrt{r\alpha + r} & \alpha \geq \omega \\ \frac{\alpha + 1}{r} & \alpha < -1 \end{cases}$ 	8
$f(\alpha) = \frac{\alpha^r - (r+1)^r}{\alpha + r} \text{ و } \frac{\alpha^r + r\alpha^r - \alpha - 1 - r\alpha - r\alpha^r}{\alpha + r} = \frac{-r\alpha - 1}{\alpha + r}$ $y = \frac{-r\alpha - 1}{\alpha + r} \rightarrow \alpha = \frac{(-ry - 1)}{y + r} \text{ و } \frac{-1 - ry}{y + r} \rightarrow f^{-1}(\alpha) = \frac{-r\alpha - 1}{r + \alpha}$ $f^{-1}(\alpha) = \frac{a\alpha + b}{r\alpha + d} \rightarrow \text{نقطة} \rightarrow \left(\frac{-r\alpha - 1}{r + \alpha} \right) \times r = \frac{-r\alpha - r}{r + \alpha} \rightarrow a = -r, b = -1$ $f^{-1}(-1) = \frac{y - 1}{r - r} = \omega$ $d = r$	9
$f(\alpha) = \frac{\alpha}{\alpha^r + r} \rightarrow y\alpha^r + y < \alpha \rightarrow \alpha^r y - \alpha + y > 0 \rightarrow \alpha < \frac{1 \pm \sqrt{1 - ry^r}}{ry}$ $R_f = \left[-\frac{1}{r}, \frac{1}{r} \right]$ $f^{-1}(\alpha) = \frac{1 \pm \sqrt{1 - r\alpha^r}}{r\alpha}, \alpha \in \left[-\frac{1}{r}, \frac{1}{r} \right]$	10