

$$f(x) = \begin{cases} x^3 - 4 & x > a \\ 12x - 2 & x \leq a \end{cases} \quad R_{(1)} \Rightarrow (a^3 - 4, +\infty) \quad R_{(2)} \Rightarrow (-\infty, 12a - 2]$$

برای اینکه این تابع پیوسته باشد باید انتهای بازه اول برابر با انتهای بازه دوم باشد. (توجه: اگر بخواهیم) $a^3 - 4 = 12a - 2$ $a^3 - 12a + 16 \geq 0$ $a^3 - 4 \geq 12a - 2$

$$\Rightarrow (a-2)(a^2+2a-1) \Rightarrow (a-2)(a+4) \geq 0 \quad a+4 \geq 0 \quad a \geq -4$$

$$f(x) = 3x + k \quad f^{-1}(2) = 4 \Rightarrow f(4) = 2 \Rightarrow \underbrace{f(4)}_2 = 12 + k \Rightarrow k = -10$$

$$f(v) = 3(v) - 10 = 11$$

$$f(f(x)) = f(3x + k) = 3(3x + k) + k \Rightarrow 9x + 3k + k \Rightarrow 9x + 4k$$

$$9x + 4k \Rightarrow 9x - 10$$

$$A(2a, a) \quad f(x) = \frac{ax}{x-1} \Rightarrow yx - y = ax \Rightarrow yx - ax = y$$

$$\Rightarrow x(y-a) = y \Rightarrow x = \frac{y}{y-a} \Rightarrow f^{-1}(x) = \frac{y}{y-a} \Rightarrow f^{-1}(2a) = a$$

$$\Rightarrow \frac{2a}{2a-a} = a \Rightarrow a = 2$$

$$f = \{(3,5), (4,7), (7,2), (9,9)\} \quad g = \{(3,2), (4,4), (9,5)\}$$

$$f \circ f^{-1} : f^{-1} = \{(5,3), (7,4), (2,7), (9,9)\} \quad \left\{ \begin{array}{l} f^{-1}(5) = 3 \\ f^{-1}(7) = 4 \\ f^{-1}(2) = 7 \\ f^{-1}(9) = 9 \end{array} \right. \quad \left\{ \begin{array}{l} f(3) = 5 \\ f(4) = 7 \\ f(7) = 2 \\ f(9) = 9 \end{array} \right.$$

$$f^{-1} \circ f \rightsquigarrow f(3) = \omega \rightsquigarrow f^{-1}(\omega) = 3 / f(4) = \nu \rightsquigarrow f^{-1}(\nu) = 4$$

$$f(5) = 2 \rightsquigarrow f^{-1}(2) = 5 / f(9) = 4 \rightsquigarrow f^{-1}(4) = 9$$

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$$f: \{(9, 9) (5, 5) (3, 4) (1, 3)\}$$

$$1) f \circ g^{-1}: g^{-1} = \{(1, 3) (4, 1) (\omega, 9)\} / g^{-1}(1) = 3 \rightsquigarrow f(3) = \omega$$

$$g^{-1}(4) = 1 \rightsquigarrow f(1) = x / g^{-1}(\omega) = 9 \rightsquigarrow f(9) = 4$$

$$f \circ g^{-1} = \{(1, \omega) (\omega, 4)\}$$

$$1) g^{-1} \circ f: f(3) = \omega \Rightarrow g^{-1}(\omega) = 9 / f(9) = 4 \Rightarrow g^{-1}(4) = 1$$

$$f(4) = \nu \Rightarrow g^{-1}(\nu) = x / f(5) = 2 \rightsquigarrow g^{-1}(2) = 5$$

$$g^{-1} \circ f = \{(3, 9) (5, 3) (9, 1)\}$$

$$f = \{(3, 4) (4, 1) (5, 0)\} \quad g = \{(1, 1) (3, 3) (4, \omega)\} \quad h = \{(1, 2) (3, 4) (4, 1) (\omega, 1)\}$$

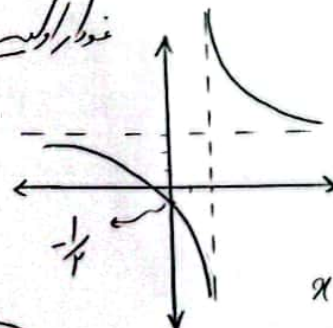
$$x=2 \Rightarrow g(2)=1 \Rightarrow f(1)=x \quad \{ \quad x=4 \Rightarrow g(4)=3 \Rightarrow f(3)=x$$

$$x=5 \Rightarrow g(5)=\omega \Rightarrow f(\omega)=x \quad \frac{h}{f \circ g^{-1}} = \emptyset \quad \left[\text{مخمر} \right]$$

$$\{(1, \frac{1}{2}) (3, \frac{1}{2}) (\omega, \frac{1}{2})\}$$

$$y = \frac{3x+1}{x-2}$$

فردا، فردا



$$\begin{cases} x=2 & \text{طابق قائم} \\ y=3 & \text{طابق افقی} \end{cases}$$

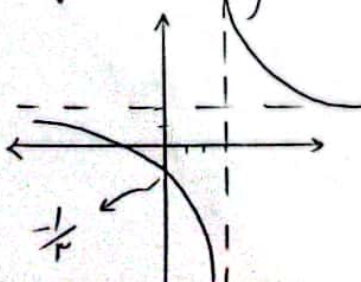
$$yx - 2y = 3x + 1$$

$$yx - 3x = 1 + 2y$$

$$x(y-3) = 1+2y \quad x = \frac{1+2y}{y-3}$$

$$\Rightarrow f^{-1}(x) = \frac{1+2x}{x-3}$$

فردا، فردا



$$\begin{cases} x=3 & \text{طابق قائم} \\ y=2 & \text{طابق افقی} \end{cases}$$

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$$f(x) = |x-1| - |x-3| \quad [a, b] \rightarrow \text{در بازه}$$

$$f(x) = \begin{cases} 2 & x < 1 \\ -2x + 4 & 1 \leq x \leq 3 \\ -2 & x > 3 \end{cases}$$

این تابع یک تابع پله‌ای است
در این بازه از تابع پله‌ای

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$$\begin{cases} a=1 \\ b=3 \end{cases} \quad f: [1, 3] \rightarrow [-2, 2] \quad / \quad \begin{aligned} y &= -2x + 4 \Rightarrow 2x = 4 - y \\ x &= \frac{4-y}{2} \Rightarrow 2 - \frac{1}{2}y = x \end{aligned}$$

$$\Rightarrow f^{-1}(x) = 2 + \frac{1}{2}x$$

$$f(x) = \begin{cases} x^3 + 4 & x \geq 1 \\ 4x - 1 & x < 1 \end{cases}$$

در این بازه از تابع پله‌ای است

$$y = x^3 + 4 \Rightarrow x^3 = y - 4 \Rightarrow x = \sqrt[3]{y-4}$$

$$\Rightarrow y = \sqrt[3]{x-4} \quad / \quad y = 4x - 1 \Rightarrow 4x = y + 1 \Rightarrow x = \frac{y+1}{4} \Rightarrow y = \frac{x+1}{4}$$

$$f^{-1}(x) = \begin{cases} \sqrt[3]{x-4} & x \geq 4 \\ \frac{x+1}{4} & x < 4 \end{cases}$$

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$$f(x) = x^2 - \frac{(x+1)^2}{x+3} \quad f^{-1}(x) = \frac{ax+b}{cx+d} \quad f^{-1}(b) = ?$$

$$\frac{x^2(x+3) - (x+1)^2}{x+3} = \frac{x^3 + 3x^2 - x^2 - 2x - 1}{x+3} \Rightarrow \frac{-x^2 - 2x - 1}{x+3}$$

$$\Rightarrow y = \frac{-x^2 - 2x - 1}{x+3} \Rightarrow yx + 3y = -x^2 - 2x - 1 \Rightarrow yx + x^2 = -3y - 1$$

$$x(y+3) = -3y - 1 \Rightarrow x = \frac{-3y-1}{y+3} \Rightarrow y = \frac{-3x-1}{x+3}$$

$$y = \frac{-3x-1}{x+3} \quad \begin{cases} a=-3 \\ b=-1 \\ c=1 \\ d=3 \end{cases} \quad f^{-1}(b) = f^{-1}(-1) \Rightarrow f^{-1}(-1) = \frac{3(-1)-1}{-1+3}$$

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$$y = \frac{x}{x^r + 1} \Rightarrow x = \frac{y}{y^r + 1} \Rightarrow x(y^r + 1) = y \quad (1.)$$

$$\Rightarrow xy^r + x = y \Rightarrow xy^r - y^r + x = 0 \Rightarrow y = \frac{+1 \pm \sqrt{(-1)^r - 4(x)(x)}}{r(x)}$$

نشر

$$\Rightarrow y = \frac{1 \pm \sqrt{1 - 4x^r}}{r x} \quad \leftarrow \text{جواب}$$