آنیاز کریم زاده - یازدهم دختر سه شنبه - تکلیف ۱۱

سوال ۱)

$f(x) = \begin{cases} x^3-4 & : x \geq a \\ 12x-20 & : x < a \end{cases}$ و $4(3x-5)$

$\rightarrow x^3-4 \geq 12x-20 \rightarrow x^3-12x+16 \geq 0$

$\rightarrow (x-2)^2(x+4) \geq 0 \rightarrow$ (−4، 2: − | + | +) $\rightarrow$ $\boxed{-4 \leq a}$

سوال ۲)

$f(x) = 3x+k = 3x-10$

$f^{-1}(2)=4 \rightarrow 3(4)+k=2 \rightarrow k=-10$

الف) $f(7) = 3(7)-10 = 11$

ب) $f(f(x)) = 3(3x-10)-10 = 9x-30-10 = 9x-40$

سوال ۳)

$f(x) = \frac{ax}{x-1} = \frac{a^2}{a-1} = 2a \rightarrow a^2 = 2a^2-2a \Rightarrow a^2-2a = a(a-2)=0$

$\begin{cases} a=0 & \text{غ ق ق} \rightarrow \frac{x}{x-0} = \frac{x}{x} \quad x \neq 0 \\ a=2 & ✓ \end{cases}$

$\rightarrow f^{-1}(x) = \frac{x}{x-a}$

$A(2a,a)$

سوال ۴)

$f(x) = \{(3,5)(4,7)(7,2)(9,6)\}$ $\quad f^{-1}(x) = \{(5,3)(7,4)(2,7)(6,9)\}$

$g(x) = \{(3,2)(8,6)(9,5)\}$ $\quad g^{-1}(x) = \{(2,3)(6,8)(5,9)\}$

الف) $fof^{-1} = \{(5,5)(7,7)(2,2)(6,6)\}$

ب) $f^{-1}of = \{(3,3)(4,4)(7,7)(9,9)\}$

ج) $fog^{-1} = \{(2,5)(5,6)\}$

د) $g^{-1}of = \{(3,9)\}$

سوال ۵)

$f(x) = \{(2,4)(4,8)(9,5)\}$

$g(x) = \{(2,1)(4,3)(9,5)\}$

$h(x) = \{(1,2)(3,4)(4,1)(5,1)\}$

$fog^{-1} = \{(5,0)(3,8)(1,4)\}$

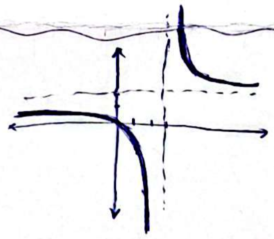
$\rightarrow \frac{h}{fog^{-1}} = \{(5,\frac{1}{0})(3,\frac{4}{8})(1,\frac{2}{4})\}$

$\rightarrow \{(3,\frac{1}{2})(1,\frac{1}{2})\}$

سوال ۶)

$f(x) = \frac{3x+1}{x-2}$ همولفینک است $\rightarrow$ معکوس پذیر

$x = \frac{3y-1}{y-2} \rightarrow \frac{2x-1}{x-3} = y \rightarrow f^{-1}(x) = \frac{2x-1}{x-3}$

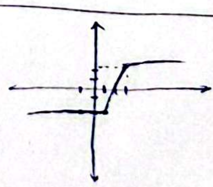


سوال ۷)

$f(x) = |x-1| - |x-3| \rightarrow$ تابع آستاری

$\begin{cases} 2 & x \geq 3 \\ 2x-4 & 1 < x < 3 \\ -2 & x \leq 1 \end{cases}$

دامنه $[a,b]$ $\rightarrow$ وارون پذیر



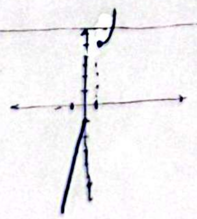
$\rightarrow [a,b] = [1,3]$

$f(x) = 2x-4$

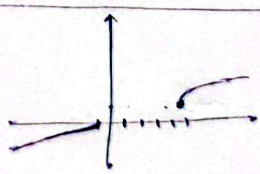
$x = 2y-4 \rightarrow y = \frac{x+4}{2} \rightarrow f^{-1}(x) = \frac{x+4}{2} \quad : 1 < x < 3$

سوال ۸)

$f(x) = \begin{cases} x^3+4 & x \geq 1 \\ 4x-1 & x \leq 0 \end{cases}$ $\rightarrow$



وارون پذیر است $\rightarrow$ وارون



$f^{-1}(x) = \begin{cases} \sqrt[3]{x-4} & : x \geq 5 \\ \frac{x+1}{4} & : x \leq -1 \end{cases}$

$x = y^3+4 \rightarrow y = \sqrt[3]{x-4} \quad : x \geq 5$

$x = 4y-1 \rightarrow y = \frac{x+1}{4} \quad : x \leq -1$

$f(x) = x^{-1} - \frac{(x+1)^2}{x+2} = \frac{x^3+2x^2-x^2-1-2x-x^2}{x+2} = \frac{-2x-1}{x+2}$ $\xrightarrow{\text{جابجایی}}$ $x = \frac{-2y-1}{y+2} \to y = \frac{-2x-1}{x+2}$

$f^{-1}(x) = \frac{ax+b}{2x+d} = \frac{-2x-2}{2x+4} \rightsquigarrow b = -2$ $\quad f^{-1}(-1) = \frac{-2(-4)-2}{-2+4} = \frac{10}{2} = \boxed{5}$

$f(x) = \frac{x}{x^2+1}$ $\xrightarrow{\text{جابجایی}}$ $\frac{y}{y^2+1} = x \to y = xy^2+x \to xy^2-y+x=0$ $\quad y = \frac{1 \pm \sqrt{1-4x^2}}{2x}$