

وقتی سه ضلع به هم وصل می‌شوند متقارن است پس آن خط از رأس می‌گذرد.

$$\text{رأس می: } \frac{-b}{2a} = 2 \Rightarrow \frac{-1}{2(a-1)} = 2 \rightarrow -1 = 4a - 4 \rightarrow 4a = 3 \rightarrow a = \frac{3}{4}$$

$$\rightarrow y = -\frac{1}{4}x^2 + x + 3 = 0 \rightarrow -x^2 + 4x + 12 = 0 \rightarrow x^2 - 4x - 12 = 0 = (x-4)(x+2) \rightarrow x=4, -2$$

طول مثبت $\rightarrow 4$

۱) معادله $mx^2 - 5x + m = 0$ دو ریشه دارد یعنی $\Delta > 0$

$$\Delta = (5)^2 - 4(m)(m) = 25 - 4m^2 \rightarrow 25 - 4m^2 > 0 \rightarrow 4m^2 < 25 \rightarrow m^2 < \frac{25}{4}$$

$$\Rightarrow -\frac{5}{2} < m < \frac{5}{2}$$

۲) $m < 0$ با توجه به اینکه معادله در بازه‌ی بین دو معرناج مثبت است پس ریشه‌ها مثبت است.

$$1 \wedge 2 \rightarrow \begin{array}{c} 0 \\ \hline -\frac{5}{2} \quad 0 \quad \frac{5}{2} \end{array} \rightarrow -\frac{5}{2} < m < 0$$

$$S = \frac{3-\sqrt{5}}{2} + \frac{1+\sqrt{5}}{2} = \frac{1+2}{2} = 2$$

$$P = \left(\frac{3-\sqrt{5}}{2}\right)\left(\frac{1+\sqrt{5}}{2}\right) = \frac{(3-\sqrt{5})(1+\sqrt{5})}{4} = \frac{9-5}{4} = \frac{4}{4} = 1$$

$$Q = a(x^2 - Sx + P) = a(x^2 - 2x + 1) \rightarrow \text{چون فراب مستقیم است} \rightarrow a=9$$

$$\Rightarrow 9x^2 - 18x + 9 = 0$$

$$3\alpha^2 + \beta^2 = 1\alpha^2 + \alpha^2 + 1\beta^2 - \beta^2 = 1\alpha - (m+2) + 1\beta - (m+2) + (\alpha^2 - \beta^2)$$

$$= 1(\alpha + \beta) + (\alpha - \beta)(\alpha + \beta) - 2(m+2)$$

$$\alpha + \beta = \frac{-(1-1)}{2} = 0, \quad \alpha - \beta = \frac{-\sqrt{\Delta}}{2a} = \frac{-\sqrt{\Delta}}{2} \rightarrow \Delta = (1)^2 - 4(m+2)(2) = 4\epsilon - 16m - 14$$

$$\rightarrow 1(0) + \left(\frac{-\sqrt{\Delta}}{2}\right)^2 - 2(m+2) = 12 \div 2, \quad 14 - \sqrt{\Delta} - (m+2) = 4$$

$$\Rightarrow \sqrt{\Delta} = 14 - 4 - m - 2 = 8 - m \rightarrow \Delta = (8-m)^2 \Rightarrow 4\epsilon - 16m - 14 = 64 + m^2 - 16m$$

$$\rightarrow m^2 - 16m + 14 = (m-8)^2 = 0 \Rightarrow m=8$$

$$y = a(x-h)^2 + k \quad \text{رأس می: } (h, k) \rightarrow y = a(x-2)^2 + 9 = a(x^2 + 4 - 4x) + 9$$

$$= ax^2 + 4a - 4ax + 9 \quad \text{عوض از سر: } 5 = 4a + 9 \Rightarrow 4a = -4 \rightarrow a = -1$$

$$\rightarrow y = -x^2 + 4x + 5$$

$$\rightarrow \begin{array}{c} -1 \quad 5 \\ \hline -1 \quad 4 \end{array} \rightarrow \text{ریشه‌ی } (-1, 5) \text{ با یی گزیده ها}$$

$$x^2 - 4x - 5 = 0 = (x-5)(x+1)$$

ریشه‌های تابع $\rightarrow 5, -1$

$\alpha\beta = \frac{9\beta}{\alpha} \Rightarrow \alpha^2 = 9 \rightarrow \alpha = \pm 3 \rightarrow \begin{cases} \alpha = -3 \rightarrow S' = \frac{-9}{-1} = 9 = -3 + \beta \rightarrow \beta = \frac{-9}{-1} + 3 = \frac{6}{-1} \rightarrow \beta > 0 \checkmark \\ \alpha = 3 \rightarrow S' = \frac{9}{3} = 3 = 3 + \beta \rightarrow \beta = \frac{9}{3} - 3 = 0 \rightarrow \beta > 0 \times \end{cases}$ $\Rightarrow \alpha = -3, \beta = \frac{6}{-1} \rightarrow \frac{1}{-3} + \frac{1}{\frac{6}{-1}} = \frac{-1}{3} + \frac{1}{6} = \frac{-2+1}{6} = \frac{-1}{6} = \frac{1}{-6}$ $\therefore \rho = \frac{9\beta}{\alpha} = \frac{9(\frac{6}{-1})}{-3} = \frac{54}{3} = 18$ $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-3+\frac{6}{-1}}{-3 \cdot \frac{6}{-1}} = \frac{-3-6}{18} = \frac{-9}{18} = \frac{-1}{2} = \frac{1}{-2}$	6
$x^r = rm - m\alpha \rightarrow \alpha^r - m\beta = rm - m\alpha - m\beta = rm - m(\alpha + \beta) = \Lambda$ $\rightarrow S = \alpha + \beta = \frac{-m}{1} = -m \rightarrow rm - m(-m) = rm + m^2 = \Lambda \rightarrow m^2 + rm - \Lambda = 0$ $= (m+2)(m-2) = 0 \Rightarrow m = 2, -2 \quad \Delta > 0 \rightarrow \Delta = m^2 + 4\Lambda = m(m+4) \quad \frac{-\Lambda}{\pm \frac{1}{2} \pm \frac{1}{2}}$ $\rightarrow m = 2, S = -m = -2$	7
$\frac{-\Delta}{\epsilon a} \rightarrow \text{مقسوم} \rightarrow \frac{-\Delta}{\epsilon a} = \Lambda \rightarrow \Delta = 14 - \epsilon(\frac{m}{2} + 1)(m) = 14 - (rm + 2\Lambda)(m)$ $= 14 - rm^2 - 2\Lambda m \rightarrow \frac{-\Delta}{\epsilon a} = \frac{rm^2 + 2\Lambda m - 14}{\epsilon m} = \Lambda$ $\rightarrow m^2 + 2\Lambda m - \Lambda = f(\epsilon m) = 14m \rightarrow m^2 - rm - \Lambda = (m-2)(m+2) = 0 \rightarrow m = 2, m = -2$ $\rightarrow -rx^r + \epsilon x + 4 = 0 \rightarrow rx^r - \epsilon x + 4 = 0 \quad \begin{cases} m = 2 \rightarrow rm^2 + 2\Lambda m + 4 \rightarrow \Delta < 0 \times \\ m = -2 \rightarrow -rm^2 + 2\Lambda m + 4 \rightarrow \Delta > 0 \checkmark \end{cases}$ $\rightarrow x^r - rx - 4 = (x-2)(x+2) = 0 \rightarrow x = \frac{2}{r} - 1$	8
$y = a(x-h)^r + k \quad (h, k) \rightarrow \text{مركز}$ $\rightarrow y = a(x-2)^r + 4 = a(x^r + \epsilon - \epsilon x) + 4 = ax^r + \epsilon a - \epsilon ax + 4$ $\rightarrow \text{مركز} = K = (\epsilon a + 4) \rightarrow \epsilon a = -2 \rightarrow a = \frac{-1}{r}$ $\rightarrow y = -\frac{1}{r}x^r + rx + \epsilon \rightarrow -\frac{1}{r}x^r + rx + \epsilon = 0$ $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} \rightarrow S' = \frac{-r}{-\frac{1}{r}} = \epsilon, \alpha\beta = \frac{\epsilon}{-\frac{1}{r}} = -\Lambda \rightarrow \frac{\epsilon}{-\Lambda} = -\frac{1}{r}$	9
$x = r \rightarrow (r)^r - (\epsilon a + \epsilon)x^r + (\epsilon a^r + 4a + 3) = 0$ $\rightarrow \epsilon - \Lambda a - 1 + \epsilon a^r + 4a + 3 = \epsilon a^r - ra - 1 = 0 \rightarrow 9a^r - 4a - 3 = (\epsilon a - 3)(\epsilon a + 1) = 0$ $\rightarrow a = 1, -\frac{1}{r}$ $a = 1 \rightarrow x^r - (\epsilon + \epsilon)x + (\epsilon + 4 + 3) = x^r - \Lambda x + 12 = 0 = (x-4)(x-2) \Rightarrow x = 4$ $a = -\frac{1}{r} \rightarrow x^r - (\epsilon(-\frac{1}{r}) + \epsilon)x + (\epsilon(-\frac{1}{r})^r + 4(-\frac{1}{r}) + 3) = 0 = x^r - \frac{\Lambda}{r}x + \frac{\epsilon}{r}$ $\rightarrow 9x^r - rx + 12 = 0 \Rightarrow (3x-4)(3x-2) = 0$ $\rightarrow x = \frac{2}{r} \rightarrow \text{مركز}$	10