

مسئله عقاری

$$y = (a-1)x^2 + x + 2, \quad \frac{-b}{2a} = 2 \Rightarrow \frac{-1}{2(a-1)} = 2$$

$$\Rightarrow -1 = 2(a-1) \Rightarrow -1 = 2a - 2 \rightarrow 2a = 1 \rightarrow a = \frac{1}{2}$$

$$y = -\frac{1}{2}x^2 + x + 2 \rightarrow -\frac{1}{2}x^2 + x + 2 = 0 \xrightarrow{\times (-2)} x^2 - 2x - 4 = 0$$

$$(x-9)(x+2) = 0 \xrightarrow{x > 0} \boxed{x = 9}$$

۲) در دو تقسیم علامت نشان می‌دهد محدوده ۲ طرف ریشه منفیست. $\Delta > 0$ و $m < 0$

$$\Delta > 0 \rightarrow 2a - 4m^2 > 0 \rightarrow 4m^2 < 2a \quad m^2 < \frac{2a}{4} \rightarrow -\frac{\sqrt{a}}{2} < m < \frac{\sqrt{a}}{2}$$

$$m < 0 \rightarrow -\frac{\sqrt{a}}{2} < m < 0$$

$$x = \frac{2 \pm \sqrt{a}}{2} = \frac{1 \pm \sqrt{a}}{1} \Rightarrow -b = 4 \rightarrow b = -4, \quad 2a = 1 \rightarrow a = \frac{1}{2}$$

$$\Delta = 20 \rightarrow b^2 - 4ac = 20$$

$$(-4)^2 - 4 \times \frac{1}{2} \times c = 20 \rightarrow 16 - 2c = 20 \rightarrow -2c = 4 \rightarrow c = -2$$

$$(2x^2 - 4x + \frac{1}{2}) \times 2 = 4x^2 - 4x + 1 = 0$$

$$y = ax^2 + bx + a \Rightarrow \frac{-b}{2a} = 2 \Rightarrow -b = 4a \rightarrow b = -4a$$

$$y(2) = 4 \rightarrow 4a + 2b + a = 4 \Rightarrow 5a + 2b = 4 \rightarrow 2a + b = 2$$

$$\rightarrow 2a - 4a = 2 \rightarrow -2a = 2 \rightarrow a = -1, \quad b = -4a = 4$$

$$y = -x^2 + 4x + 1$$

$$x^2 + 4x + 1 = 0 \quad x^2 - 4x - 1 = 0$$

$$(x-1)(x+1) = \frac{-1}{-1 \pm 1} = \frac{-1}{-2}$$

از بالای محور می‌باشد، یعنی که مثبت شود.

$$r m^r - \lambda m + m + r = 0 \quad , \quad r \alpha^r + \beta^r = 1 \quad (r = \alpha^r + \beta^r + \underbrace{\lambda \alpha^r}_{\lambda \alpha - (m+r)} = 1) \quad (K)$$

$$S = r, \quad \rho = \frac{m+r}{r}$$

$$\underbrace{\alpha^r + \beta^r}_{S=r} + \lambda \alpha - (m+r) = 1 \quad \Rightarrow \quad 1 - \cancel{\lambda} \frac{m+r}{r} = 1 - (m+r) + \lambda \alpha - m - r = 1 \quad (3)$$

$$S = r - r\rho$$

$$\cancel{r m^r} - m - r + \lambda \alpha - m - r = \cancel{r} \quad \Rightarrow \quad -r m - \cancel{r} + \lambda \alpha = \cancel{r} \quad \Rightarrow \quad m = r \alpha \quad *$$

$$r \alpha^r - \lambda \alpha + r \alpha + r = 0 \quad \Rightarrow \quad r \alpha^r - r \alpha + r = 0 \quad \stackrel{r}{\Rightarrow} \quad \alpha^r - r \alpha + 1 = 0$$

$$\Rightarrow (\alpha - 1)^r = 0 \quad \Rightarrow \quad \alpha = 1 \quad \star \quad \Rightarrow \quad m = r \times 1 = r \quad \boxed{r}$$

$$r m^r - v m + r \beta = 0 \quad (4)$$

$$\cancel{\alpha \beta} = \frac{r \beta}{r} = \alpha^r = r \quad \Rightarrow \quad \alpha = \pm r / \cancel{\alpha + \beta = \frac{v}{r}} \quad (5)$$

$$\alpha + \beta = \frac{v}{r} \quad \left\{ \begin{array}{l} \rightarrow \alpha = r \rightarrow r + \beta = \frac{v}{r} \rightarrow \beta = -\frac{r}{r} \quad \text{و } \beta \in \mathbb{R} \text{ غير ممكن} \\ \alpha = -r \rightarrow -r + \beta = -\frac{v}{r} \rightarrow \beta = r - \frac{v}{r} \rightarrow \beta = \frac{r}{r} \quad \checkmark \end{array} \right.$$

$$\alpha = -r, \quad \beta = \frac{r}{r} \quad \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha \beta} = \frac{-\frac{v}{r}}{-r} = + \frac{v}{r}$$

$$r m^r + m m - r m = 0 \quad \Rightarrow \quad \beta^r + m \beta - r m = 0 \quad -m \beta = \beta^r - r m \quad \star \quad (V)$$

$$\alpha^r - m \beta = 1 \quad \star \quad \Rightarrow \quad \underbrace{\alpha^r + \beta^r}_{S=r-r\rho} - r m = 1 \quad \Rightarrow \quad (-m)^r - r(m) - r m = 1 \quad (6)$$

$$m^r + r m - r m = m^r + r m - 1 = 0 \quad \Rightarrow \quad (m+r)(m-r) = 0 \quad \text{و } m = -r$$

$$m \rightarrow -r \rightarrow \Delta < 0 \quad \text{و } \beta \in \mathbb{R} \text{ غير ممكن}$$

$$\Rightarrow r \rightarrow \Delta > 0 \quad \checkmark \quad \Rightarrow \quad r m^r + r m - r = 0 \quad \boxed{S = -r}$$

$$f(m) = m^2 + 2m + \frac{m+1}{2}, \quad -\frac{\Delta}{\epsilon a} = 1 \quad (1)$$

$$\Delta = 14 - 2m \left(\frac{m+1}{2} \right) = 14 - 2m^2 - 2m = 0$$

$$\frac{-\Delta}{\epsilon a} = \frac{2m^2 + 2m - 14}{2m} = \frac{m^2 + 1m - 7}{m} = 1 \quad (5)$$

$$m^2 + 1m - 7 = 14m \Rightarrow m^2 - 13m - 7 = 0 \Rightarrow (m-15)(m+2) = 0$$

$$m = \begin{cases} 15 \\ -2 \end{cases} \rightarrow \text{بما أن } m \in \mathbb{N} \rightarrow 15 \text{ فقط}$$

$$f(m) = -2m^2 + 2m + 4 \Rightarrow$$

$$-2m^2 + 2m + 4 = 0 \Rightarrow m^2 - m - 2 = 0 \Rightarrow (m-2)(m+1) = 0$$

$$\Rightarrow m = 2 \text{ أو } -1 \quad K > \text{عدد صحيح} \Rightarrow \boxed{K = 2}$$

$$\text{مثال } y = an^2 + bn + c, \quad -\frac{b}{2a} = 2, \quad -\frac{\Delta}{\epsilon a} = 9 \quad (9)$$

$$-\frac{b}{2a} = 2 \Rightarrow -b = 4a \Rightarrow b = -4a$$

$$y(1) = 9 \Rightarrow a + b + c = 9 \Rightarrow a + 4b = 9 \Rightarrow a + b = 1$$

$$\rightarrow a + b = 1 \quad a - 4a = 1 \Rightarrow -3a = 1 \Rightarrow a = -\frac{1}{3}$$

$$b = -4a = -4 \times -\frac{1}{3} = \frac{4}{3} \Rightarrow y = -\frac{1}{3}n^2 + \frac{4}{3}n + c$$

$$\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{1}{-1} \Rightarrow \boxed{-1}$$

(10)

$$n^2 - (ka + c)n + (ka^2 + 4a + c) = 0$$

نمایند که برای هر دو مقدار $a > 0$ و $c > 0$

(5)

$$n=7 \rightarrow k - (ka + c)(7) + ka^2 + 4a + c$$

$$k - 7ka - 7c + ka^2 + 4a + c = 0 \rightarrow ka^2 - 7ka - 6c + 4a = 0$$

$$a = 19 - \frac{1}{k}$$

$$\Rightarrow \left\{ \begin{array}{l} \rightarrow a=1 \rightarrow n^2 - 7n + 12 = 0 \rightarrow (n-3)(n-4) = 0 \rightarrow n=3, 4 \end{array} \right. \quad (7)$$

$$\rightarrow a = -\frac{1}{k} \rightarrow n^2 - \frac{1}{k} + \frac{k}{k} = 0 \xrightarrow{\times k} kn^2 - n + 1 = 0$$

$$\Delta = 1 - 4k = 17 \quad \frac{1 \pm \sqrt{17}}{2} = \frac{1 \pm k}{4} = 7 \quad \left(\frac{1}{k} \right)$$