پاسخنامه تشریحی تکلیف شماره ..۱۴...... کلاس .... شاربیا جعفری..

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$y = -x^2 + bx + 3$ $\qquad max = 7$ $\qquad b = ?$

$y_s = \frac{\Delta}{-4a} = 7 \Rightarrow \frac{b^2+12}{-4 \times -1} = 7 \qquad b^2 + 12 = 28$

$b^2 = 16$

$b = \pm 4$

$if\ b = 4 \rightarrow y = -x^2 + 4x + 3$ ✓

$if\ b = -4 \Rightarrow y = -x^2 - 4x + 3$ ✓

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صفرهای توابع

الف) $f(x) = x^4 + 2x^2 + 3 \qquad x^2 = t \qquad t^2 + 2t + 3 \qquad \Delta = 4 - 12 < 0$

ریشه حقیقی ندارد.

ب) $f(x) = (4 - x^2)^2 - 2(4 - x^2) - 15 \qquad 4 - x^2 = t \qquad t^2 - 2t - 15 = 0 \qquad (t-5)(t+3) = 0$

$t = 5 \rightarrow 4 - x^2 - 5 = 0 \Rightarrow x^2 = -1$ غ ق ق

$t = -3 \rightarrow 4 - x^2 + 3 = 0 \Rightarrow x^2 = 7 \Rightarrow \boxed{x = \sqrt{7}}$

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$4x^2 - 16x + m = 0$ یکی از جواب‌ها دو واحد از جواب دیگر بزرگتر $\qquad m = ? \qquad \alpha = ? \qquad \beta = ?$

α و $\alpha + 2$

$S = \frac{-b}{a} = \frac{16}{4} = 4 \Rightarrow \alpha + \alpha + 2 = 4 \qquad 2\alpha = 2 \qquad \alpha = 1 \rightarrow \alpha = 1,\ \beta = 3$

$P = \frac{c}{a} = \frac{m}{4} \qquad 3 = \frac{m}{4} \Rightarrow \boxed{m = 12}$

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$3x^2 - 6x + m - 1 = 0 \qquad 2\alpha - \beta = 4 \qquad m = ?$

$2\alpha - \beta = 4 \qquad \beta = 2\alpha - 4$

$S = \frac{-b}{a} = \frac{6}{3} = 2 \Rightarrow 2\alpha - 4 + \alpha = 2 \Rightarrow 3\alpha = 6 \Rightarrow \alpha = 2$

$\beta = 2\alpha - 4 \rightarrow \beta = 0$

$P = \frac{m-1}{3} = 0 \qquad m - 1 = 0$

$\boxed{m = 1}$

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$-mx^2 + 4x + m^2 - 2 = 0 \qquad m = ? \qquad \alpha, \beta \qquad \beta = \frac{1}{\alpha}$

$\alpha \times \frac{1}{\alpha} = 1$

$\frac{m^2 - 2}{-m} = 1 \qquad m^2 - 2 + m = 0 \qquad (m+2)(m-1)$

$m = -2 \rightarrow 2x^2 + 4x + 2 = 0 \rightarrow \Delta = 0$ دارای ریشه مضاعف ✗

$m = 1 \rightarrow -x^2 + 4x - 1 = 0$

$\hookrightarrow \Delta > 0 \Rightarrow \boxed{m = 1}$

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$x^2 - mx + 3 = 0 \qquad \alpha\beta^2 = 4 \qquad m = ?$

$P = \frac{c}{a} = \frac{3}{1} = \alpha.\beta$

$\alpha\beta^2 = 4 \quad (\alpha.\beta)\beta = 4 \quad 3\beta = 4 \quad \beta = \frac{4}{3}$

$\alpha\beta = 3 \xrightarrow{\beta = \frac{4}{3}} \frac{4}{3} \times \alpha = 3 \quad \alpha = 3 \times \frac{3}{4} = \frac{9}{4}$

$S = \frac{-b}{a} = m \Rightarrow \alpha + \beta = m \Rightarrow \frac{4}{3} + \frac{9}{4} = m \Rightarrow m = \frac{16+27}{12} = \boxed{\frac{43}{12}}$

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$x^2 - 4x + m = 0 \qquad \alpha, \beta \qquad \alpha = 3\beta \qquad m = ?$

$S = \frac{-b}{a} = 4 \Rightarrow 3\alpha + \alpha = 4 \Rightarrow 4\alpha = 4 \Rightarrow \alpha = 1$

$P = \alpha.\beta \xrightarrow{\beta = 3\alpha} P = 3\alpha^2 = m \xrightarrow{\alpha = 1} \boxed{m = 3}$

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ریشه α: $x^2 - 7x + 2 = 0 \qquad \alpha^3 + \frac{8}{\alpha^3} = ? \qquad \alpha^2 + 2 = 7\alpha$

$(\alpha + \frac{2}{\alpha})^3 = \alpha^3 + \frac{8}{\alpha^3} + 3\alpha^2 \times \frac{2}{\alpha} + 3\alpha \times \frac{4}{\alpha^2} \Rightarrow (\alpha + \frac{2}{\alpha})^3 = \alpha^3 + \frac{8}{\alpha^3} + 6\alpha + \frac{12}{\alpha}$

$(\frac{\alpha^2+2}{\alpha})^3 = \alpha^3 + \frac{8}{\alpha^3} + \frac{6\alpha^2+12}{\alpha} \xrightarrow{\alpha^2 + 2 = 7\alpha} (\frac{7\alpha}{\alpha})^3 = \alpha^3 + \frac{8}{\alpha^3} + \frac{6(\alpha^2+2)}{\alpha} \; 7\alpha$

$7^3 - 42 = \alpha^3 + \frac{8}{\alpha^3}$

$343 - 42 = \boxed{301}$

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بازیکنی توپی را به سمت دروازه شوت می‌کند. ارتفاع توپ (h) در زمان t از رابطه‌ی $h(t) = -10t^2 + 50t$ بدست می‌آید. ارتفاع ماکسیمم توپ و زمانی که توپ به زمین برخورد می‌کند را بدست آورید.

$h(t) = -10t^2 + 50t$

$h_{max} = y_s \qquad x_s = \frac{-50}{2 \times (-10)} = \frac{5}{2} \Rightarrow y_s = -10 \times \frac{25}{4} + 50 \times \frac{5}{2} = \frac{-125+250}{2} = \frac{125}{2}$ = $\boxed{62/5}$

برخورد به زمین $\rightarrow -10t^2 + 50t = 0 \qquad t(-10t + 50) = 0$

$t = 0$ یا $\boxed{5}$

لحظه شروع

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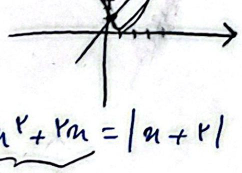
حل با روش هندسی

الف) $(x-1)^2 = 2x + 1$ — نقاط برخورد $\left|\begin{smallmatrix}0\\1\end{smallmatrix}\right.$ و $\left|\begin{smallmatrix}4\\9\end{smallmatrix}\right.$



$x^2 + 1 - 2x - 2x - 1 = 0$ ($-4x$)

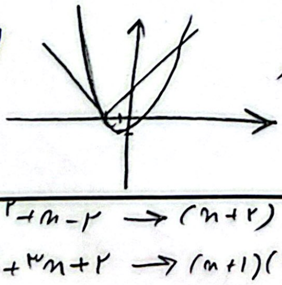
$x^2 - 4x = 0$

$x = 0, 4$

ب) $x^2 + 2x = |x + 2|$

$x_s = -1$

$y_s = -1$



۲ نقطه برخورد

$x^2 + 2x - x - 2 = x^2 + x - 2 \rightarrow (x+2)(x-1) \qquad x = -2, 1, -1$

$x^2 + 2x + x + 2 = x^2 + 3x + 2 \rightarrow (x+1)(x+2) \qquad \text{if } x = -1 \quad x^2 + 2x = |x+2| \qquad x = -2, 1$

$-1 \neq 1 \quad \times$