تمیز و خوانا نوشته شود

۱۸٫۷۵

لطفا با خودکار

(۱)

$m = \frac{y_2 - y_1}{x_2 - x_1} =$

(۲)

لطفا با خودکار

$y - y_1 = m(x - x_1)$

$y - 2 = -4(x - 4)$

$m = \frac{-2-2}{5-4} = -4$

$y - 2 = -4x + 16$

$y = -4x + 18$

الف)

ب)

$\frac{a}{a'} = \frac{b}{b'} \neq \frac{c}{c'} \rightarrow \begin{vmatrix} a & b \\ a' & b' \end{vmatrix} = 0$

$y - 2 = -3(x - 4)$

$y = -3x - \frac{1}{2}$

$y - 2 = -3x + 12 \rightarrow y = -3x + 14$

ج)

$3y = -x + 1 \rightarrow y = -\frac{x}{3} + \frac{1}{3}$

$m_1 = -\frac{1}{3} \rightarrow m_2 = -\frac{1}{-\frac{1}{3}} = 3$

$y - y_1 = m(x - x_1) \rightarrow y - 2 = 3(x - 4) = y - 2 = 3x - 12$

$y = 3x - 10$

---

(۵)

$\tan\alpha = m \rightarrow \tan\frac{\pi}{3} = \sqrt{3} \rightarrow y - 2 = \sqrt{3}(x - 4)$

$y - 2 = \sqrt{3}x - 4\sqrt{3} \rightarrow y = \sqrt{3}x - 4\sqrt{3} + 2$

(۲) الف) $|AB| = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2} \to \sqrt{(7-3)^2+(-1-4)^2} = \sqrt{25} = 5$

ب) $\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}} \Rightarrow \frac{|3^{12}(2)+4^{13}(3)-3|}{\sqrt{9+16}} = \frac{15}{5} = 3$

(۳) الف) $\begin{cases} 2x+3y=7 \to 4x+6y=14 \\ 4x+6y=8 \to 4x+6y=8 \end{cases}$

$ax+by = \frac{c+c'}{2} \to 4x+6y = \frac{14+8}{2} \to 4x+6y = 11$

ب) $\begin{cases} 4x+6y=14 \\ 4x+6y=8 \end{cases} \to 4x+6y=11$

$d = \frac{|c-c'|}{\sqrt{a^2+b^2}} = \frac{|14-11|}{\sqrt{16+36}} = \frac{3}{\sqrt{52}}$

(۴) $\frac{|ax+by-d|}{\sqrt{a^2+b^2}} = \frac{|ax'+by'-c'|}{\sqrt{(a')^2+(b')^2}} \Rightarrow \frac{|3x-2y-1|}{\sqrt{13}} = \frac{|2x+3y-4|}{\sqrt{13}}$

$\to \begin{cases} 3x-2y-1 = 2x+3y-4 \to x-5y=-3 \\ 3x-2y-1 = -2x-3y+4 \to 5x+y=5 \end{cases}$

(۵) $\tan\alpha = \left|\frac{m-m'}{1+mm'}\right|$

$\begin{cases} y+3x=3 \to y=-3x+3 \to m'=-3 \\ y-2x=5 \to y=2x+5 \to m=2 \end{cases}$

$\tan\alpha = \left|\frac{2+3}{1-6}\right| = \left|\frac{5}{-5}\right| = |-1| = 1$

$\alpha = 45°$

(۶) الف) $AB = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2} = \sqrt{(4-(-2))^2+(-5-3)^2} = \sqrt{100} = 10$

ب) $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = M\left(\frac{-5+3}{2}, \frac{4-2}{2}\right) \Rightarrow M(-1,1)$

(۷) الف) $x_G = \frac{x_1+x_2+x_3}{3} = \frac{-10-2+3}{3} = -\frac{9}{3} = -3$

$y_G = \frac{y_1+y_2+y_3}{3} = \frac{-12+3+1}{3} = -\frac{9}{3} = -3$

$G(-3,-3)$

ب) $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \Rightarrow \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ -2 & 3 & 1 \\ -10 & -12 & 1 \end{vmatrix} = \frac{9-1+24+30+24}{2} = 105$

$S = \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ -2 & 3 & 1 \\ -10 & -12 & 1 \end{vmatrix} = 48$

(٨) $y = \frac{2x+1}{4x-3}$

ا) $y \to -y \longrightarrow -y = \frac{2x+1}{4x-3}$

ب) $x \to -x \longrightarrow y = \frac{-2x+1}{-4x-3}$

ج) $\begin{cases} x \to y \\ y \to x \end{cases} \longrightarrow x = \frac{2y+1}{4y-3} \longrightarrow y = \frac{3x+1}{4x-3}$

د) $\begin{cases} x \to -y \\ y \to -x \end{cases} \longrightarrow -x = \frac{-2y+1}{-4y-3} \longrightarrow y = [redacted] \Rightarrow y = -\frac{3x-1}{4x+2}$

---

(٩) ا) $\begin{matrix} x' = x-2 \\ y' = y+3 \end{matrix} \longrightarrow \begin{matrix} x = x'+2 \\ y = y'-3 \end{matrix} \longrightarrow y'-3 = \frac{2(x'+2)+1}{(x'+2)-3} \longrightarrow y' = \frac{2x'+5}{x'-1} + 3$

$y' = \frac{2x'+5+3x'-3}{x'-1}$

$y' = \frac{5x'+2}{x'-1}$

ب) $\begin{matrix} x' = x-3 \\ y' = y-2 \end{matrix} \longrightarrow \begin{matrix} x = x'+3 \\ y = y'+2 \end{matrix}$

$\longrightarrow y'+2 = \frac{2(x'+3)+1}{(x'+3)-3} \longrightarrow y' = \frac{2x'+7}{x'} - 2 \longrightarrow \frac{2x'+7-2x'}{x'} = y' = \frac{7}{x'}$

---

(١٠)

ا) $\begin{cases} 3x+4y = 2 \longrightarrow 3x+4y = 2 \\ x-5y = 1 \longrightarrow -3x+15y = -3 \end{cases} \times -3$

$19y = -1 \longrightarrow y = -\frac{1}{19}$

$x = \frac{14}{19}$

ب) $\begin{cases} 3x+4y = 2 \\ x-5y = 1 \end{cases}$

$x = \frac{\begin{vmatrix} b & c \\ b' & c' \end{vmatrix}}{\begin{vmatrix} a & b \\ a' & b' \end{vmatrix}} = \frac{\begin{vmatrix} 4 & 2 \\ -5 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & 4 \\ 1 & -5 \end{vmatrix}} = \frac{4-(-10)}{-15-4} = \frac{14}{-19} = \frac{14}{19}$

$y = \frac{\begin{vmatrix} a & c \\ a' & c' \end{vmatrix}}{\begin{vmatrix} a & b \\ a' & b' \end{vmatrix}} = \frac{\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & 4 \\ 1 & -5 \end{vmatrix}} = \frac{3-2}{-15-4} = -\frac{1}{19}$