

سوال 1

$A(4, -4), B(2, 1)$

شیب AB از A و B : $\frac{1 - (-4)}{2 - 4} = -\frac{5}{2} \rightarrow m = -\frac{5}{2}$ $y = ax + b \rightarrow y = -\frac{5}{2}x + b$

$(4, -4) \rightarrow -4 = -\frac{5}{2}(4) + b$

$b = \frac{5}{2}$

$y = -\frac{5}{2}x + \frac{5}{2}$

$M_{AB} = (\frac{4+2}{2}, \frac{-4+1}{2}) = (3, -\frac{3}{2})$

$A(4, 1), B(-4, 7), C(-1, 3)$

$\overline{BC} = \sqrt{(-4-(-1))^2 + (7-3)^2} = 5$

شیب BC : $m = \frac{7-3}{-4-(-1)} = -\frac{4}{3}$ $y = -\frac{4}{3}x + b$

$(-1, 3) \rightarrow 3 = -\frac{4}{3}(-1) + b \rightarrow b = \frac{10}{3}$ $y = -\frac{4}{3}x + \frac{10}{3}$

$\overline{AH} = \frac{|1 + \frac{4}{3} - \frac{10}{3}|}{\sqrt{1 + \frac{16}{9}}} = \frac{\frac{1}{3}}{\frac{5}{3}} = \frac{1}{5}$

$\overline{BC} - \overline{AH} = 5 - \frac{1}{5} = \frac{24}{5}$

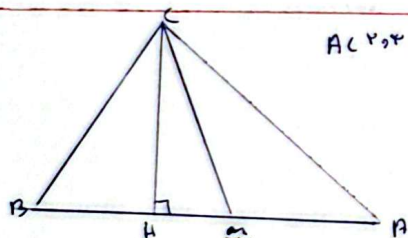
$xy - x = \sqrt{x^2}$ $xy - x = 1 \rightarrow xy - x = 1$

$xy = ax + b \rightarrow xy - ax = b \rightarrow a = 1$

مساحت دایره: $\frac{|1 - 1|}{\sqrt{1 + (-1)^2}} = \frac{0}{\sqrt{2}} = 0$ $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

شعاع دایره: $0, 4\sqrt{2}$

$S_{\text{دایره}} = \pi r^2 \rightarrow \pi \times (\frac{4\sqrt{2}}{10})^2 = \frac{16\pi}{25}$



$A(4, 3), B(8, 1), C(-1, -3)$

نام نقطه M از H = نام نقطه M از H و M از H و M از H

$m_{AB} = \frac{1-3}{8-4} = -\frac{1}{2} \rightarrow m_{CH} = 1$

$y = -\frac{1}{2}x + b$ $(-1, -3) \rightarrow -3 = -\frac{1}{2}(-1) + b \rightarrow b = -\frac{7}{2}$ $y = -\frac{1}{2}x - \frac{7}{2}$

$(\frac{4+8}{2}, \frac{3+1}{2}) = (6, 2)$

$\overline{MH} = \frac{|12 - 3 + 4|}{\sqrt{5}} = \frac{13}{\sqrt{5}} = \frac{13\sqrt{5}}{5}$

$(m+1)x - (2m-5)y + 1 = 0$ $(3m-4)x + (5-m)y - 1 = 0$

مستقیم b و b' موازی: $m \times m' = -1$

$\frac{-(m+1)}{-(3m-4)} \times \frac{-(5-m)}{5-m} = -1 \Rightarrow \frac{-m-1}{3m-4} = \frac{5-m}{5-m}$

$5m^2 - 13m + 1 = 0$

$\Delta = 169 - 20 = 149$

حاصل جذری قابل قبول نیست $\Delta < 0$

$$\begin{aligned} AB: x+2y &= 4 \\ AC: y &= 2x-1 \\ BC: x+y &= 5 \end{aligned} \quad \begin{aligned} &\xrightarrow{A: (1,1)} \begin{aligned} 2y &= 4-x \\ y &= 2x-1 \\ 4-x &= 2x-1 \end{aligned} \\ &\xrightarrow{B: (1,1)} \begin{aligned} x+2y &= 4 \\ x+y &= 5 \end{aligned} \end{aligned}$$

$$\begin{aligned} y &= 2x-1 \\ y &= -x+5 \\ 2x-1 &= -x+5 \\ 3x &= 6 \Rightarrow x=2, y=3 \end{aligned}$$

مقدار $\angle C$ را پیدا کنید

$$A(1,1) \quad B(2,3) \quad C(5,-1)$$

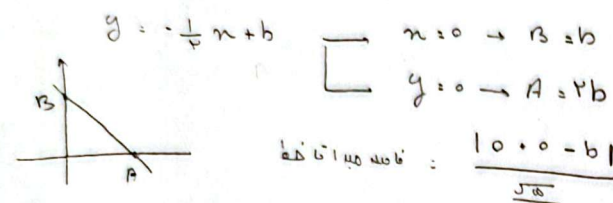
$$\Rightarrow m_{BC} = \frac{3-(-1)}{2-5} = \frac{4}{-3} = -\frac{4}{3}$$

$$y = -\frac{4}{3}x + b \Rightarrow -1 = -\frac{4}{3}(5) + b \Rightarrow b = \frac{17}{3} \Rightarrow y = -\frac{4}{3}x + \frac{17}{3}$$

$$AH = \frac{|1+1-5|}{\sqrt{5}} = \frac{3}{\sqrt{5}}$$

$$M = \left(\frac{\frac{5}{2} + 5}{2}, \frac{\frac{5}{2} - 1}{2} \right) = \left(\frac{15}{4}, \frac{3}{4} \right) \Rightarrow AM = \sqrt{\left(1 - \frac{15}{4}\right)^2 + \left(1 - \frac{3}{4}\right)^2} = \frac{5\sqrt{5}}{4}$$

$$\frac{AM}{AH} = \frac{\frac{5\sqrt{5}}{4}}{\frac{3}{\sqrt{5}}} = \frac{25}{12}$$



$$AB = \sqrt{(2b-0)^2 + (0-b)^2} = \sqrt{4b^2 + b^2} = \sqrt{5b^2} = b\sqrt{5}$$

مساحت مثلث $OAB = \frac{1}{2} \times 2b \times b = b^2$

مساحت مثلث $OAB = \frac{1}{2} \times AB \times OM = \frac{1}{2} \times b\sqrt{5} \times OM$

$b^2 = \frac{1}{2} \times b\sqrt{5} \times OM \Rightarrow OM = \frac{2b}{\sqrt{5}}$

$$A(10,0) \quad B(0,5)$$

$$AB = \sqrt{100 + 25} = \sqrt{125} = 5\sqrt{5}$$

$$\begin{aligned} 1: m_1 y - \frac{1}{2}x &= 2m \Rightarrow y = \frac{1}{m}x + 4 \\ 2: y + m &= x + 2 \Rightarrow y = (1-m)x + 2 \end{aligned} \quad \begin{aligned} &\rightarrow \text{محل تقاطع در } x=0 \\ &\text{محل تقاطع در } y=2 \end{aligned}$$

$$\begin{aligned} 1: y=0 &\Rightarrow x = -4 \\ 2: y=0 &\Rightarrow x = -2 \end{aligned}$$

$$S_{\text{مثلث}} = \frac{1}{2} \times \text{طول پایه} \times \text{ارتفاع} \Rightarrow \frac{1}{2} \times \frac{1}{m} \times \left| \frac{m}{1-m} - \frac{2}{1-m} \right| = \frac{1}{2}$$

$$\frac{m}{1-m} - \frac{2}{1-m} = \frac{1}{m} \Rightarrow \frac{m-2}{1-m} = \frac{1}{m} \Rightarrow (m-2)(1-m) = 1-m^2 \Rightarrow m^2 - 4m + 2 = 0 \Rightarrow m = 2$$

$$\frac{m}{1-m} - \frac{2}{1-m} = -\frac{1}{m} \Rightarrow \frac{m-2}{1-m} = -\frac{1}{m} \Rightarrow (m-2)(1-m) = -1-m^2 \Rightarrow m^2 - 4m + 2 = 0 \Rightarrow m = -2 \pm \sqrt{5}$$

مساحت $m=2$ و $m=-2 \pm \sqrt{5}$

$$S = \frac{1}{2} \times \frac{1}{m} \times \left| \frac{m}{1-m} - \frac{2}{1-m} \right| = \frac{1}{2}$$

$$y = 2x - 4 \quad M(2, -2)$$

محل تقاطع در $x=2$ $\Rightarrow A(2, 0)$

$$m_{AB} = \frac{y_A' + 4}{2} = 2 \Rightarrow y_A' = 0$$

$$y_A' = -2 \Rightarrow y = 2x - 4$$

$$y = x + 4 \quad A(1, 2)$$

$$m_{AA'} = -1 \xrightarrow{(1,2)} y = -x + b \Rightarrow 2 = -1 + b \Rightarrow b = 3 \Rightarrow y = -x + 3$$

$$\begin{aligned} y &= x + 4 \\ y &= -x + 3 \end{aligned} \Rightarrow x = -1, y = 3 \Rightarrow M(-1, 3)$$

$$m_B = \frac{y_B + y_{A'}}{2} \Rightarrow -1 = \frac{1 + y_{A'}}{2} \Rightarrow y_{A'} = -3 = a$$

$$m_B = \frac{y_A + y_{A'}}{2} \Rightarrow 3 = \frac{2 + y_{A'}}{2} \Rightarrow y_{A'} = 4 = b$$

$$y = b - a = 4 - (-3) = 7$$