یازدهم دفتر B   تکلیف ۱۸   ۲۰   آرتمیس اکبری

سوال ① =>

$\frac{\text{طول}}{\text{عرض}} = \frac{5}{4}$ $x =$ طول ، $y =$ عرض

$\frac{x}{y} = \frac{5}{4} \longrightarrow y = 4k ,\ x = 5k \longrightarrow S_1 = 4k \times 5k = 20k^2$

$\frac{x'}{y} = \frac{1+\sqrt{5}}{2} \xrightarrow{y = 4k} x' = 4k\left(\frac{1+\sqrt{5}}{2}\right) \longrightarrow S_2 = 4k \times 4k\left(\frac{1+\sqrt{5}}{2}\right) = 16k^2\left(\frac{1+\sqrt{5}}{2}\right)$

$\frac{S_2}{S_1} = \frac{16k^2\left(\frac{1+\sqrt{5}}{2}\right)}{20k^2} = \frac{4}{5}\left(\frac{1+\sqrt{5}}{2}\right) = \boxed{\frac{2}{5}(1+\sqrt{5})}$

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سوال ② =>

$\frac{\text{قطر}}{\text{طول}} = \frac{1+\sqrt{5}}{2}$ $x =$ طول $y =$ عرض   قطر $= \sqrt{x^2+y^2}$

$\frac{\sqrt{x^2+y^2}}{x} = \frac{1+\sqrt{5}}{2} \longrightarrow \frac{x^2+y^2}{x^2} = \frac{(1+\sqrt{5})^2}{4} \xrightarrow[\text{صورت}]{\text{تفضیل خرج در}} \frac{y^2}{x^2} = \frac{(1+\sqrt{5})^2 - 4}{4}$

$$\boxed{\frac{x^2}{y^2} = \frac{2}{1+\sqrt{5}}}$$

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سوال ③ =>

$3a + \sqrt{2a^2+4a} = 2$, $\frac{a+1}{a} = ?$

$3a + \sqrt{2a^2+4a} = 2 \longrightarrow \sqrt{2a^2+4a} = 2 - 3a$

$2a^2 + 4a = 4 - 12a + 9a^2$

$7a^2 - 16a + 4 = 0$

$a^2 - 16a + 28 = 0 \to (a-14)(a-2) = 0$

$a = \frac{14}{7} = 2 \longrightarrow$ در معادله صدق نمی‌کند

$\boxed{a = \frac{2}{7}}$ $\frac{a+1}{a} = \frac{\frac{2}{7}+1}{\frac{2}{7}} = \frac{\frac{9}{7}}{\frac{2}{7}} = \boxed{\frac{9}{2}}$

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سوال ④ =>

$\frac{\sqrt{x+1}}{\sqrt{x-1}+3} - \frac{\sqrt{x+1}}{3-\sqrt{x-1}} = \frac{x-1}{\sqrt{x-1}}$

$\frac{\sqrt{x+1}\,(3-\sqrt{x-1}) - \sqrt{x+1}\,(\sqrt{x-1}+3)}{9-(x-1)} = \frac{\overbrace{x-1}^{(\sqrt{x-1})^2}}{\sqrt{x-1}} \longrightarrow \frac{3\sqrt{x+1} - \sqrt{x^2-1} - \sqrt{x^2-1} - 3\sqrt{x+1}}{10-x} = \sqrt{x-1}$

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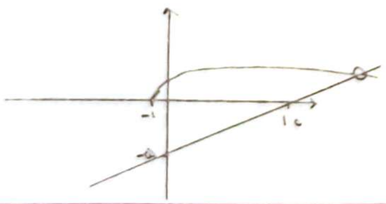
$\frac{-2\sqrt{x^2-1}}{10-x} = \sqrt{x-1} \to -2\cancel{\sqrt{x-1}}\sqrt{x+1} = (10-x)\cancel{\sqrt{x-1}} \to -2\sqrt{x+1} = 10-x$

$* \quad \sqrt{x-1} \neq 0$ $\sqrt{x+1} = \frac{x}{2} - 5$



$\boxed{\text{یک ریشه مثبت دارد}}$

سوال ⑤

$\frac{1}{\sqrt{2-x}+2} - \frac{1}{2-\sqrt{2-x}} = \frac{2-x}{5\sqrt{2-x}} \to \frac{2-\sqrt{2-x}-\sqrt{2-x}-2}{\underbrace{4-2+x}} = \frac{\overbrace{2-x}^{(\sqrt{2-x})^2}}{5\sqrt{2-x}} = \frac{\sqrt{2-x}}{5}$

$\hookrightarrow x \neq 2 \to \sqrt{2-x} \neq 0$

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$-10\sqrt{2-x} = \sqrt{2-x}\,(2+x) \to -10 = 2+x \to x = -12$

$\boxed{\text{جواب مثبت ندارد}}$

سوال ۶

$$\frac{1}{x^2}+\frac{1}{(1-x)^2}=\frac{160}{9} \longrightarrow \left(\frac{1}{x}+\frac{1}{1-x}\right)^2-2\left(\frac{1}{x(x+1)}\right)=\frac{160}{9}$$

$$\longrightarrow \left(\frac{1}{x(1-x)}\right)^2-2\left(\frac{1}{x(1-x)}\right)=\frac{160}{9} \xrightarrow{\frac{1}{x(1-x)}=t} t^2-2t=\frac{160}{9} \longrightarrow t^2-2t+1=\frac{169}{9}$$

$$(t-1)^2=\frac{169}{9}$$

$$t-1=\frac{13}{3} \rightarrow t=\frac{16}{3}$$

$$t-1=\frac{-13}{3} \rightarrow t=\frac{-10}{3}$$

$$\frac{1}{x(1-x)}=\frac{16}{3} \rightarrow -x^2+x-\frac{3}{16}=0$$

$$S_1=1$$

$$\frac{1}{x(1-x)}=\frac{-10}{3} \rightarrow -x^2+x+\frac{3}{10}=0$$

$$S_2=1$$

$$\boxed{S_1+S_2=2}$$

سوال ۷

$$\sqrt{x+\sqrt{-x^3+4x^2+25x-100}}+\sqrt{x^2\sqrt{-x^2+6x-8}}=x+2$$

$$\sqrt{-x^2+6x-8} \rightarrow -x^2+6x-8\geq 0 \rightarrow x^2-6x+8\leq 0 \rightarrow (x-2)(x-4)\leq 0$$

x		2		4	
	$+$	0	$-$	0	$+$

$\rightarrow [2,4]$ (I)

$$\sqrt{-x^3+4x^2+25x-100} \rightarrow -x^3+4x^2+25x-100\geq 0 \Rightarrow x^3-4x^2-25x+100\leq 0 \rightarrow (x^3-25x)-(4x^2-100)\leq 0$$

$$x(x^2-25)-4(x^2-25)\leq 0$$

$$(x-5)(x+5)(x-4)\leq 0$$

x		-5		4		5	
	$-$	0	$+$	0	$-$	0	$+$

$(-\infty,-5]\cup[4,5]$ (II)

$\mathrm{I}\cap\mathrm{II} \Rightarrow x=4$

$$\sqrt{4+\sqrt{-64+64+100-100}}+\sqrt{16+\sqrt{-16+24-8}}=4+2 \rightarrow \boxed{6=6}$$

$x=4$ ق ق

یک جواب

سوال ۸

$$y=|x+2|+|x-1| \quad , \quad 3y+x=17$$

	-2		1	
$-x-2-x+1$		$x+2-x+1$		$x+2+x-1$
$-2x-1$		3		$2x+1$

$$3=\frac{-x+17}{3}$$

$\rightarrow x=8$ غ ق ق

$$\begin{cases} y+2x=-1 \\ 3y+x=17 \end{cases}$$

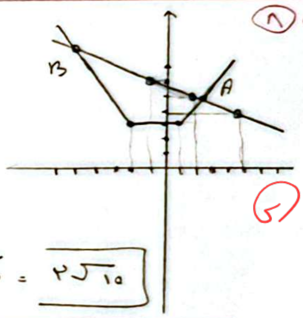
$$-5x+20$$

$x=-4 , y=7 \rightarrow B$

$$\begin{cases} y-2x=+1 \\ 3y+x=17 \end{cases}$$

$$7x=14$$

$x=2 , y=5 \Rightarrow A$

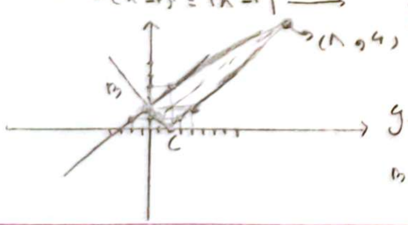


$$AB=\sqrt{\Delta x^2+\Delta y^2} \rightarrow \sqrt{36+4}=\sqrt{40}=\boxed{2\sqrt{10}}$$

$$y=\sqrt{x^2-4x+4} \quad , \quad y=\frac{1}{2}x+2$$

$$\sqrt{(x-2)^2}=|x-2| \longrightarrow$$

	2	
$-x+2$		$x-2$



$$y+x=2$$
$$-y+\frac{x}{2}=-2$$
$$\frac{3}{2}x=0 \rightarrow x=0 , y=2$$

$$y-x=-2$$
$$-y+\frac{x}{2}=-2$$
$$-\frac{1}{2}x=-4$$
$$x=8 , y=6 \Rightarrow A$$

فاصله A از خط $y=-x+2$: $\dfrac{|6+8-2|}{\sqrt{2}}=\dfrac{12\sqrt{2}}{2}=6\sqrt{2}$

$$BC=\sqrt{4+4}=2\sqrt{2}$$

$$S=\frac{2\sqrt{2}\times 6\sqrt{2}}{2}=\boxed{12}$$

سوال ۱۰

دستور $=\dfrac{1}{x}$ ، لودهار $=\dfrac{1}{x+9}$ ، هردو $=\dfrac{1}{20}$

$$\frac{1}{x}+\frac{1}{x+9}=\frac{1}{20} \longrightarrow 20(x+9)+20x=x(x+9)$$

$$20x+180+20x=x^2+9x$$

$$x^2-31x-180=0$$

$$(x-36)(x+5)=0$$

$\boxed{x=36}$ $\quad x=-5$ غ ق ق